Ministerio del Ambiente
y de los Recursos
Naturales Renovables

# HIDROLOGY AND GEOMORPHOLOGY

Submitted to Water Resources
Research #2

# A RAINFALL-RUNOFF ANALYSIS OF THE GEOMORPHOLOGIC IUH

by

Juan B.Valdés, Yolanda Fiallo and Ignacio Rodríguez-Iturbe

Graduate Program in Hydrology and Water Resources
Universidad Simón Bolívar, Caracas, Venezuela

# ABSTRACT

The instantaneus unit hydrograph derived from the geomorphologic characteristics of a basin and a dynamic component, the velocity of the discharge when the peak occurs, was presented in Rodríguez-Iturbe and Valdés (1978). To analyze this geomorphologic IUH in real world basins a geomorphologic study was carried out on several basins in Venezuela and Puerto Rico. The geomorphologic IUH for each basin was then compared with the IUH's derived from the discharge hydrograph produced by a rain - fall-runoff model of the same basins. The effect that the nonlinearities of the rainfall-runoff model have on the derivation of the IUH are analyzed and further experiments are carried out in which the IUH is derived under constant velocity conditions. The geomorphologic IUH's and the ones obtained in the experiments are remarkably identical in all the basins analyzed.

## 1. Introduction

The response function of a basin as a function of its geomorphologic parameters was derived in a companion paper (Rodríguez-Iturbe and Valdés, 1978). In this paper, the ability of the geomorphologic IUH to represent the response function of real world basins is analyzed. In particular, the representation of the non-linearities of the response function of a basin as a linear IUH with a time varying velocity is analyzed. To do this the basins are modelled with a non-linear rainfall-runoff model from which output the IUH is derived.

## 2. Description of the Basins

Two very different regions are considered in the experiments. The first region is in the northern part of Puerto Rico with very high levels of annual precipitation whereas the second area is in the central part of Venezuela near the city of Barquisimeto where the annual levels of precipitations are only one third of those of the Puerto Rico area.

### Indio Basin Area (Puerto Rico)

The two basins selected within the Indio basin area are the Morovis basin with 13 km$^2$ and the Unibon basin with 23 km$^2$. Both of those rivers are tributaries to the Indio River which itself drains into the Cibuco River, which goes into the Atlantic Ocean near the city of Vega Baja.

The Morovis and Unibon Basins are located between the 66° 30' and 66° 15' of latitude and 18° 15' N and 18° 30' N of longitude. The general layout of the two basins and their location in the island is shown in Figure 1a.

Mean annual precipitation over the basin is between 60 and 90 inches and is relatively uniform along the year as it is shown in Figure 1b.

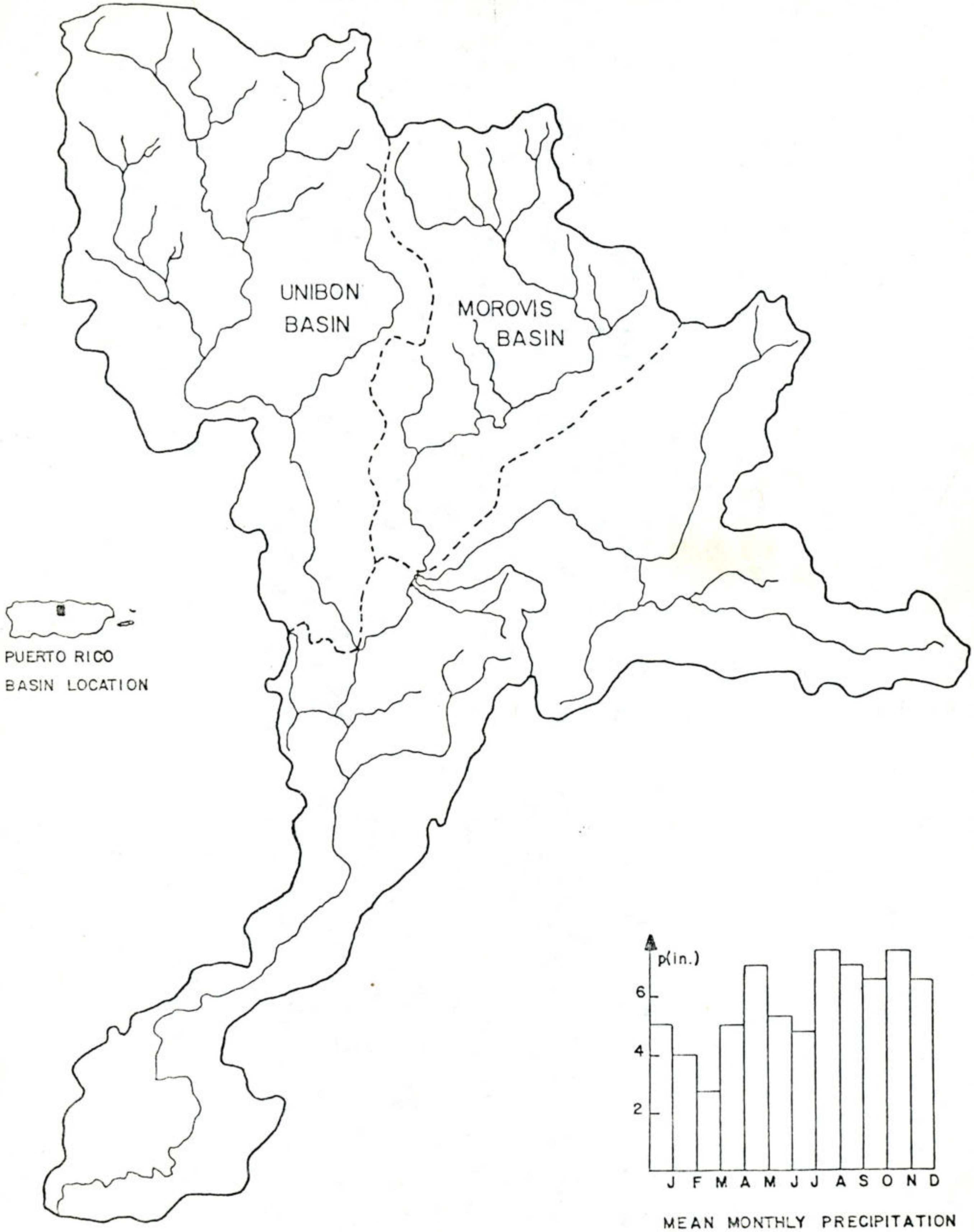


FIGURE 1. Indio basin (Puerto Rico) with the Morovis and the Unibon sub-basins.

## Mamon Basin Area (Venezuela)

The Mamon River basin is located on the central part of Venezuela with an area of 103 $km^2$. The general layout of the basin is shown in Figure 2. The climate is almost of the desert type with a mean annual precipitation of 500 mm. (20 in.) which is less than one third of the precipitation on the Puerto Rico area, but it is also distributed uniformly throughout the year. The potential annual evapotranspiration is very high (2000 mm). The entire basin is characterized by low round hills extensively eroded and some alluvial planes among them. The complete lack of a forest cover had made the slopes very unstables and easy to erosionate.

Due to the climate characteristic, the streams, although almost dry most of the time, have large floods with very high velocities which cause large erosions in the slopes. Figure 3 shows a stream in the basin in which the almost vertical slopes due to large floods may be seen.

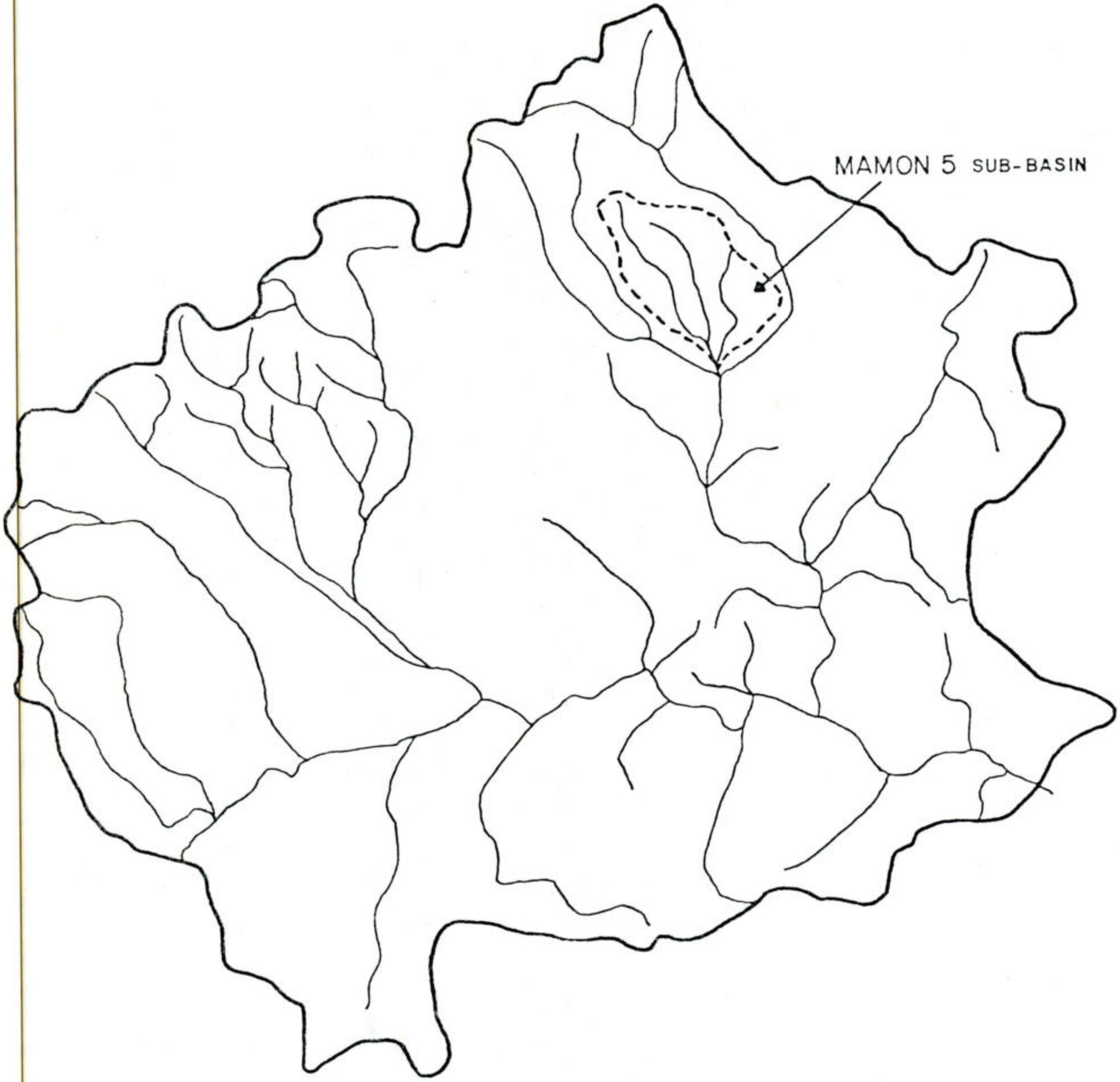
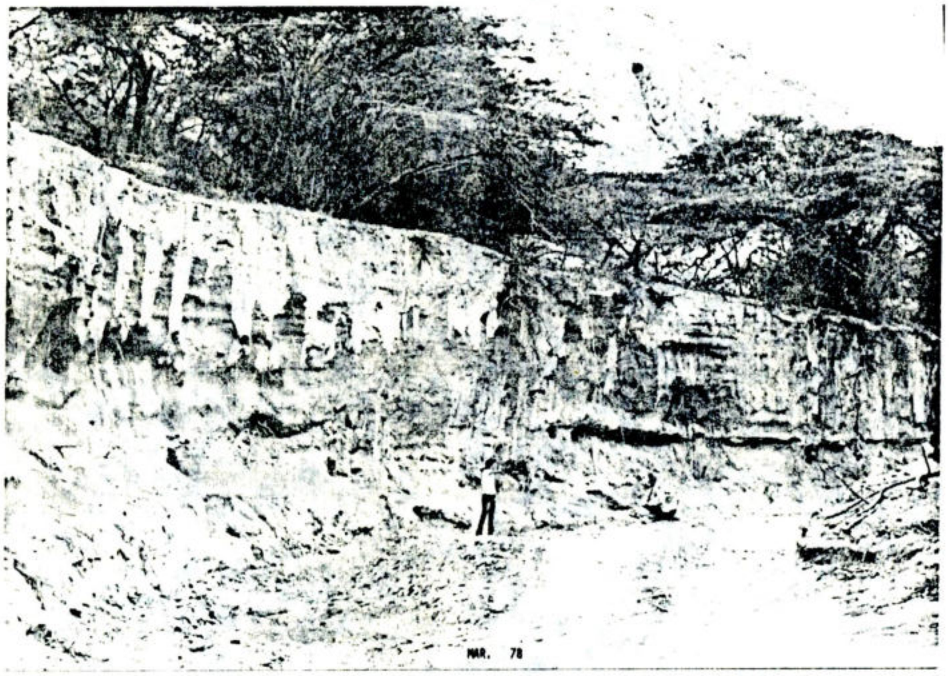


Figure 2. Mamon basin (Venezuela).



MAR. 78

FIGURE 3. Typical cross section of the streams in the Mamon basin.

## Geomorphologic Analysis of the Basins.

The Horton's numbers $R_A$, $R_B$ and $R_L$ which quantitatively represent the Horton's laws of basin drainage composition were estimated for all basins using Strahler's ordering procedure. A visual fit was performed in each one of the diagrams and in the case of $R_B$ the lines were drawn through the point $N_\Omega = 1$.

The Horton diagrams for all basins are displayed in Fig.4 and Horton numbers for the four basins are given in Table 1. As it can be seen from the Table, all numbers fall into the limits usually found in nature. The entire Indio basin, which was not used in this study, had an area of extremely pervious limestone and thus the $R_A$ number did not fall into those limits. This is a case of geologic control and Horton laws do not apply in those cases.

The bifurcation ratios, $R_B$, of the four basins are very similar. This is not the case, however, with the other two ratios. The Venezuelan basins have a very high drainage density, almost ten times that of the basins in Puerto Rico. This is somewhat reflected in the values of $R_A$ and $R_L$ which are larger for the Puerto Rican basins than for the Venezuelan basins. As mentioned earlier the Mamon basin have very large floods with very high velocities although the streams are most of the times almost dry. These high velocities, the complete lack of forest cover and the soil type tend to produce a very high value of drainage density.

The basins order ranged from 6 for the whole Mamon basin

| Basin | Area(km$^2$) | $L_\Omega$(km) | $R_B$ | $R_A$ | $R_L$ | order |
|---|---|---|---|---|---|---|
| Morovis (Puerto Rico) | 13.0 | 8.0 | 3.2 | 5.0 | 2.7 | 3 |
| Unibon (Puerto Rico) | 23.0 | 8.6 | 4.0 | 5.6 | 2.8 | 3 |
| Mamón (Venezuela) | 103.0 | 12.25 | 3.5 | 4.5 | 2.1 | 6 |
| Mamón5 (Venezuela) | 3.15 | 3.59 | 3.3 | 3.8 | 2.5 | 4 |

TABLE 1. Geomorphologic parameters of the four basins analyzed.

to 3 for the basins in Puerto Rico. The 6th order basin was represented in both the rainfall-runoff experiments and in the derivation of the geomorphologic IUH as a 4th order basin using in the latter case the relations of $L_\Omega$ shown in the paper by Rodríguez-Iturbe and Valdés (1978). Similarly the Mamon 5 basin was assumed to be of third order.

## 3 Description of the Experiments

To test the geomorphologic IUH of Rodríguez-Iturbe and Valdés (1978) controlled experiments were carried out to obtain the IUH's of the four basins described earlier under different dynamic conditions. A very detailed modeling of each basin is made in which every stream segment is modelled as an individual segment in the rainfall-runoff model developed originally for Schaake (1970). This model is based on the continuity equation and on the kinematic wave approximations to the equations of motion; the reasons for choosing this particular model are its great simplicity and the extensive familiarity that the authors had with the model throughout the last years. Furthermore, models based on the kinematic wave have been applied to several basins in Puerto Rico and Venezuela with very good results. See for example Resource Analysis, Inc(1976)and Rodríguez -Iturbe (1974) among others.

The rainfall-runoff model allows for spatial and temporal variability of the precipitation and it gives the option to represent the infiltration either by the Horton equation or the method of the Soil Conservation Service. Nevertheless, the use of several storm events will introduce uncertainties about the spatial and temporal distribution of the storm and about the infiltration losses. For these reasons, the controlled experiments used storms of constant intensity and uniformely distributed throughout the basin. The derivation of the storm is adopted such that it is larger than the time to equilibrium. Further, to avoid the problem of how to deal with infiltration losses and since the unit hydrograph theory deals with effective rainfall the basins are assumed imperveous throughout the controlled experiments.

## IUH Derivation through a Rainfall-Runoff Model

The outflow discharge Q(t), for a storm of duration $t_r$ ($>t_e$) and constant intensity $i_0$, is given by the IUH theory as

$$Q(t) = \int_0^t h(\tau)\ i\ (t-\tau)\ d\tau \tag{1a}$$

or

$$Q(t) = \int_0^t h(t-\tau)\ i\ (t)\ d\tau \tag{1b}$$

where

$$i(t) = i_0 \qquad \text{if} \quad t < t_r$$

$$= 0 \qquad \text{if} \quad t > t_r$$

$h(\tau)$ = ordinate of the IUH

The derivative of Q(t) gives the ordinates of the IUH. For a storm of infinite duration, $t_r = \infty$, the derivative of Equation (1) is

$$\frac{dQ(t)}{dt} = \frac{d}{dt} \int_0^t h(\tau) \cdot i_0\ d\tau$$

and using Liebniz's rule

$$\frac{dQ(t)}{dt} = i_0 \cdot h(t) \tag{2}$$

To derive the IUH for a storm lasting longer than the time to equilibrium but less than infinite, the precipitation input is defined as following (Schaake, 1978):

$$i(t) = i_0 \left[ u(t) - u(t - t_r) \right] \tag{3}$$

where

$$u(t) = 0 \qquad t < 0$$

$$= 1 \qquad t \geq 0$$

and IUH's may be obtained from both the rising limb and the descending limb of the hydrograph. Thus

$$\frac{dQ(t)}{dt} = i_0 \cdot h(t) \qquad \text{for } t < t_r \tag{4a}$$

$$\frac{dQ(t)}{dt} = i_0 \left[ h(t) - h(t - t_r) \right] \quad \text{for } t > t_r \tag{4b}$$

The derivatives in Equation (4a) are equal to zero when steady state is reached and two expressions are obtained from Equation (4), one to compute the IUH ordinates from the rising limb of the hydrograph, i.e.

$$\frac{dQ(t)/dt}{i_0} = h_1(t) \tag{5a}$$

and another one for the IUH ordinates computes from the descending limb of the discharge hydrograph , i.e.

$$-\frac{dQ(t)/dt}{i_0} = h_2\ (t) \tag{5b}$$

This procedure is illustrated by Figure 5 and was applied to the four basins mentioned earlier for several storm intensities going from 1cm/hr. to 6 cm/hr. and the IUH's derived for them are shown in Figures 6,7,8 and 9. As it can be seen from the Figures, the IUH's are quite different both for a same storm but computed from the rising or descending limbs of the outflow hydrograph and also for different storm intensities. This is because the rainfall-runoff model which is based on the kinematic wave is a non-linear representation of catchment runoff. Thus for the same storm the IUH computed from the rising limbs takes longer to reach the peak discharge and it reachs a higher value than the one obtained from the descending limbs. Further, although the IUH ordinates were normalized by the storm intensity the IUH's computed from the rising limb for different storm intensities are not the same. This is also due to the non-linearities of the rainfall-runoff model. The IUH's computed from the rising limbs for different storm intensities for the four basins are shown in Fig.10.

The comparison of the IUH's obtained from the rainfall-runoff with the geomorphologic IUH is not possible at this stage because it is needed to define a velocity which will allow to compute the geomorphologic IUH. This velocity, of course, changes during the storm from zero at the beginning until it reaches a maximum at equilibrium time, remains constant until the storms ends and then start to decrease. Because the velocity variations are larger in the part of the S-curve where

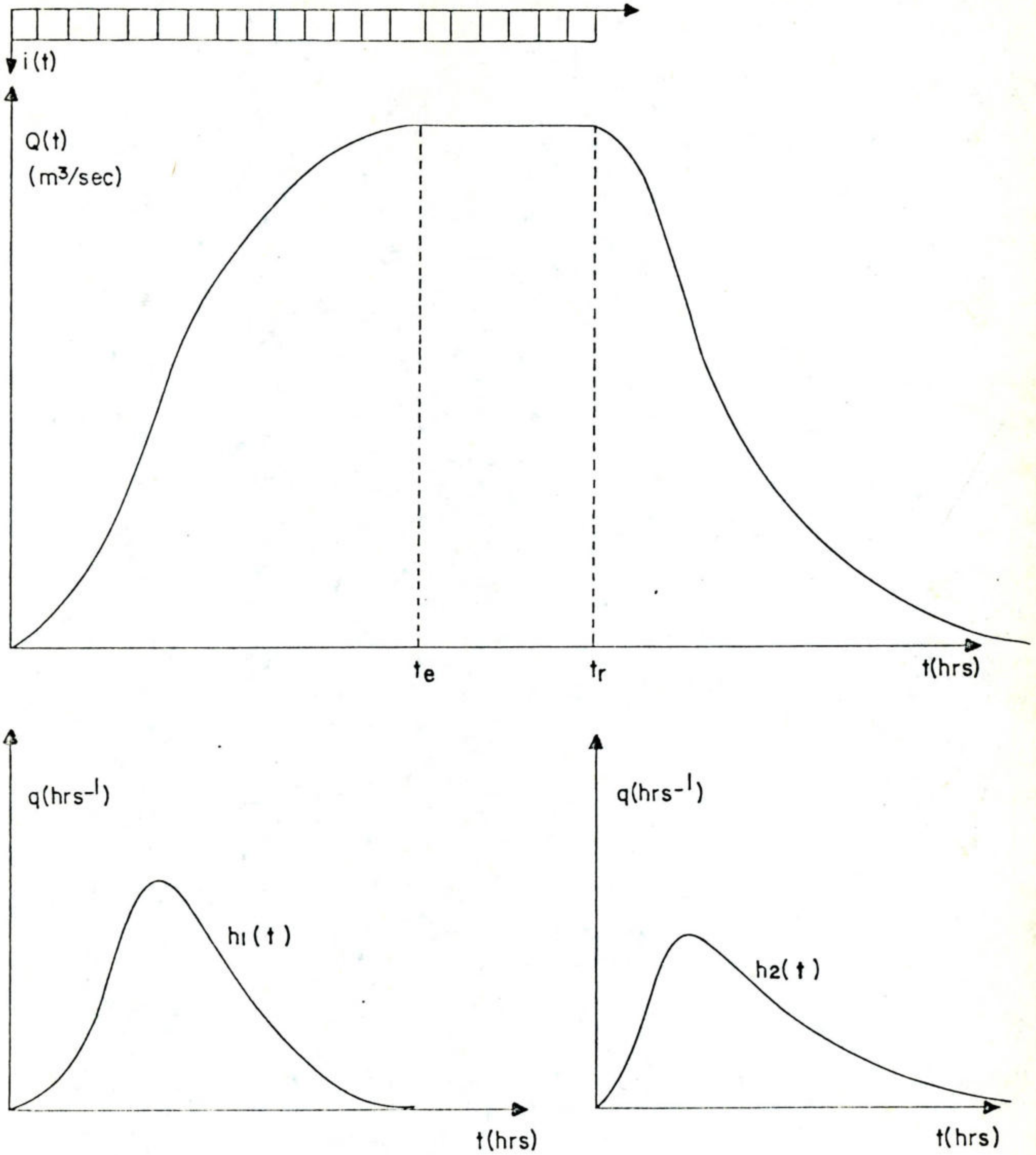


FIGURE 5. Description of the rainfall-runoff experiments to derive the response function of a basin.

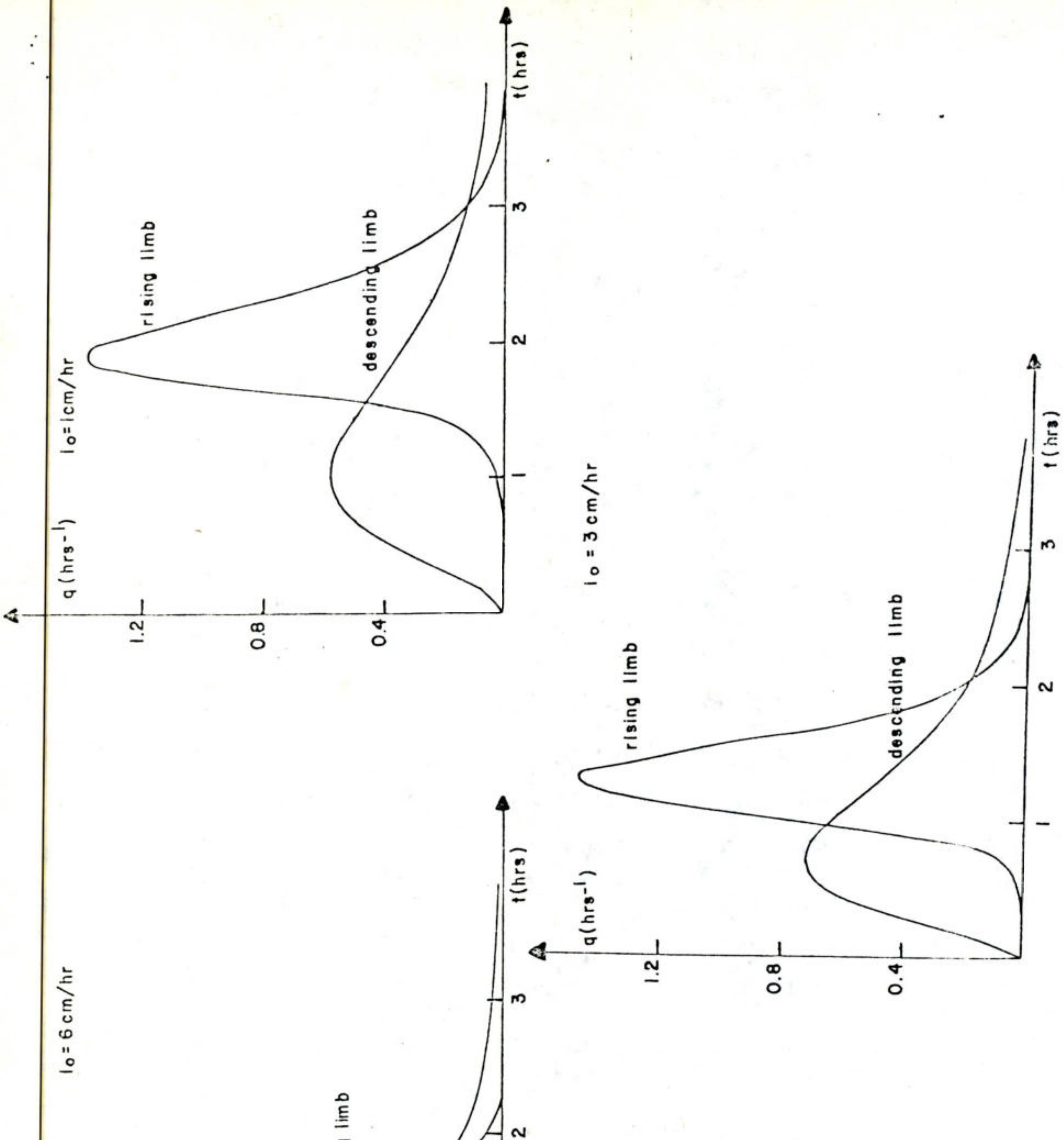


FIGURE 6. Rainfall-runoff derived IUH's for the Unibon basin for different storm intensities.

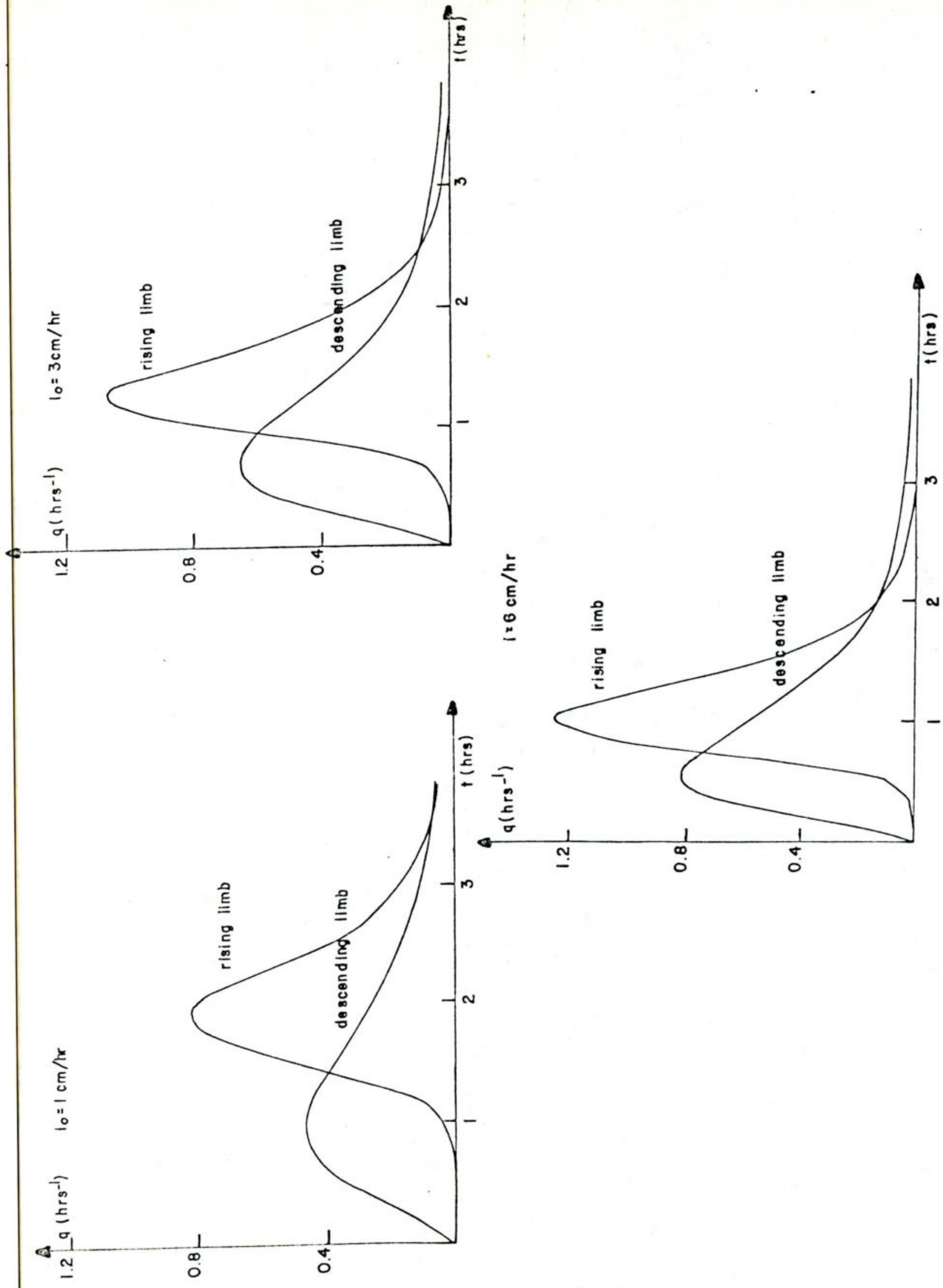


FIGURE 7. Rainfall-runoff derived IUH's for the Morovis basin for different storm intensities.

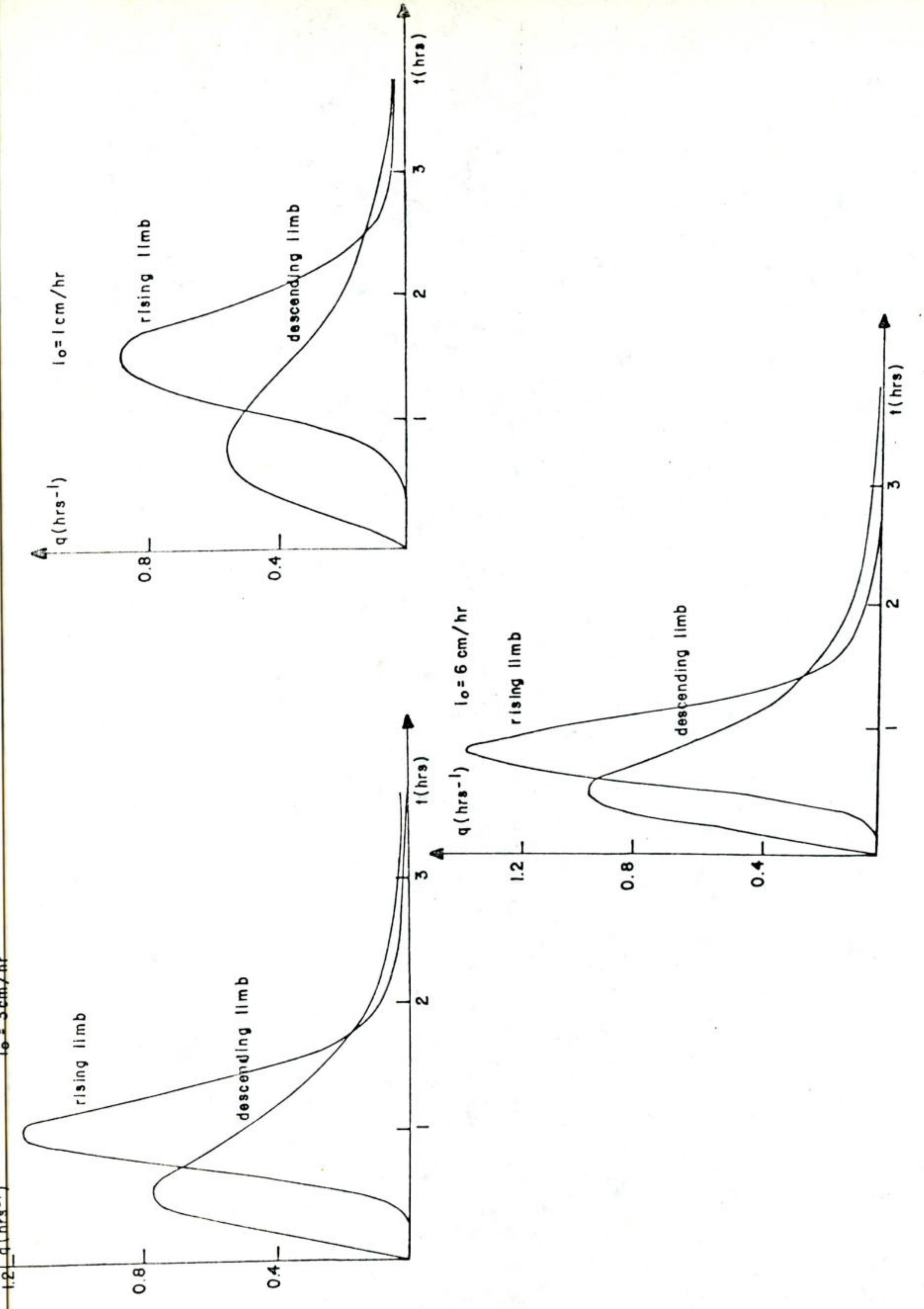


FIGURE 8. Rainfall-runoff derived IUH's for Mamon basin for different storm intensities.

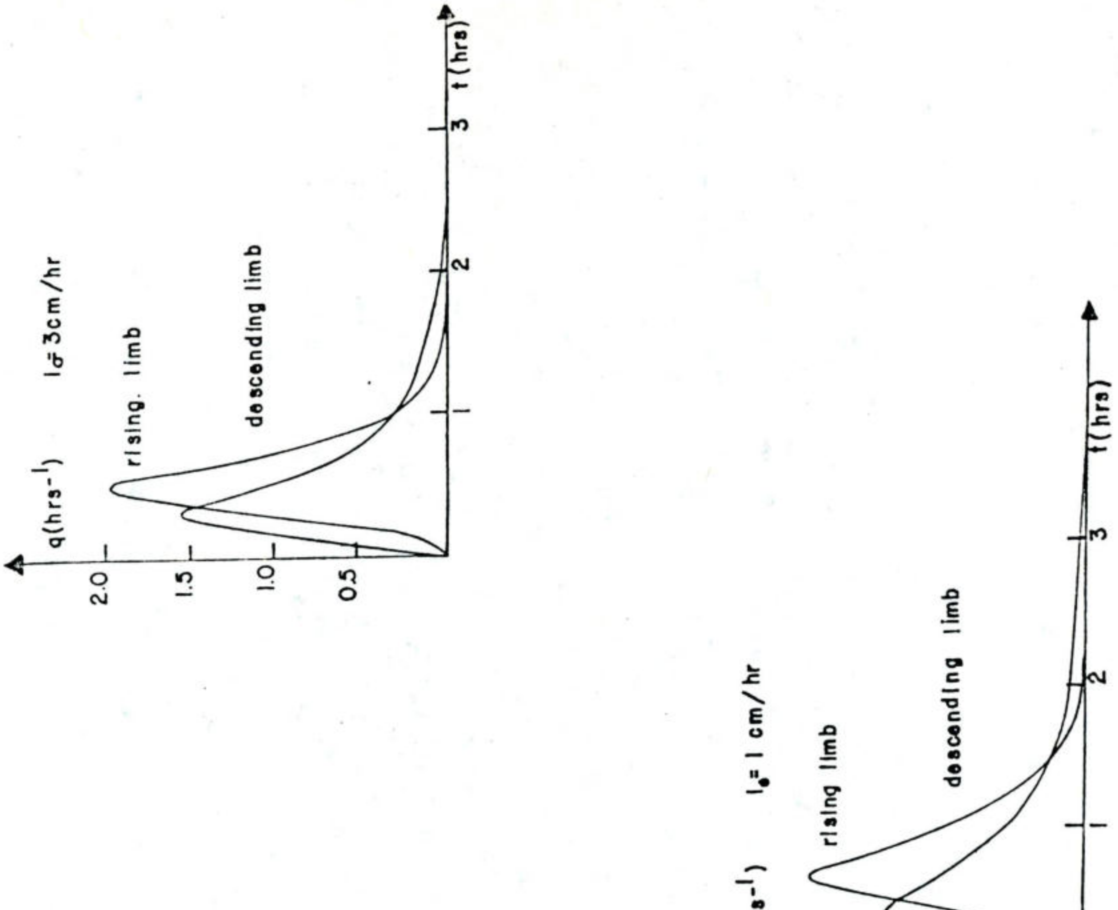


FIGURE 9. Rainfall-runoff derived IUH's for the Mamon 5 basin for different storm intensities.

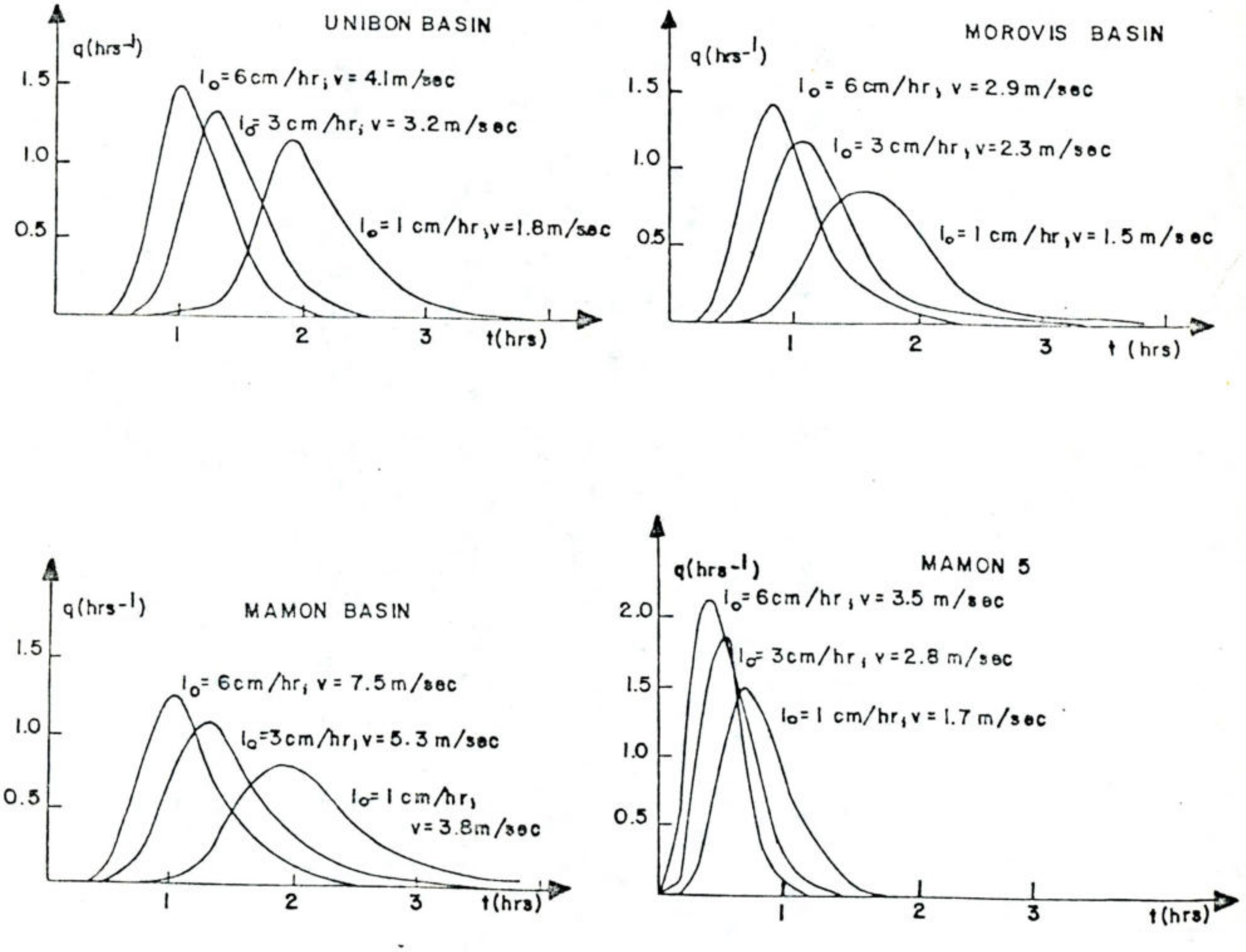


FIGURE 10. Variations in the response function of the four basins as a function of the storm intensities.

the rising limb IUH is computed, than for the case of the descending limb IUH, the comparison of the geomorphologic IUH with a given velocity, say that at the time of equilibrium, is going to give a better fit with the descending limb IUH than for the rising limb IUH. Although this kinematic component, v, allows the geomorphologic IUH to vary from storm to storm and also through a given storm representing as a linear time-variant response function the response of a non-linear system, the purpose of the controlled experiments was to derive an IUH from the rainfall-runoff model in which the velocity was kept constant.

In a conversation with the authors, Dr. John Schaake, of the U.S. National Weather Service, suggested the realization of a so called "jack-up" experiment in which after the discharge reaches a platteau at the time of equilibrium the intensity of rainfall is increased a further 10 percent of the original intensity, $i_0$.

This experiment is illustrated in Figure 11. Due to the small increase in rainfall intensity the velocity among the "jack-up" part of the hydrograph remains practically constant and equal to the velocity at equilibrium time. The interesting point is that the IUH's computed from rising limbs and the descending limbs of the "jack-up" hydrograph are practically the same for all intensities and for all basins. Figures 12, 13, 14 and 15 show examples of these comparisons. However, as it can be seen from Figure 16 the IUH's from the "jack-up" part of the hydrograph are not the same for different storm intensities for the same basin. This is because, although the IUH's are derived under a

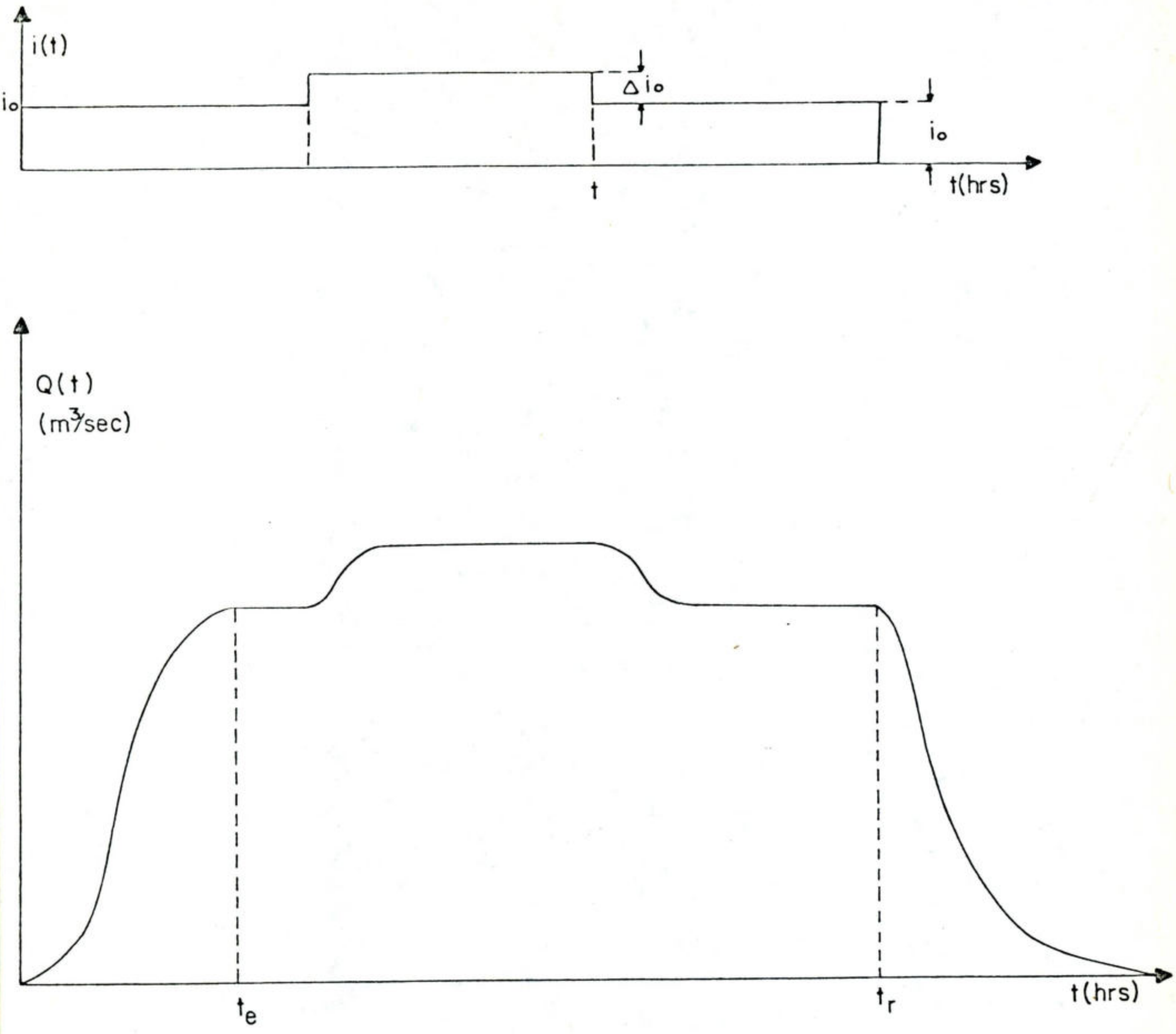


Figure 11. Description of the "jack-up" experiment.

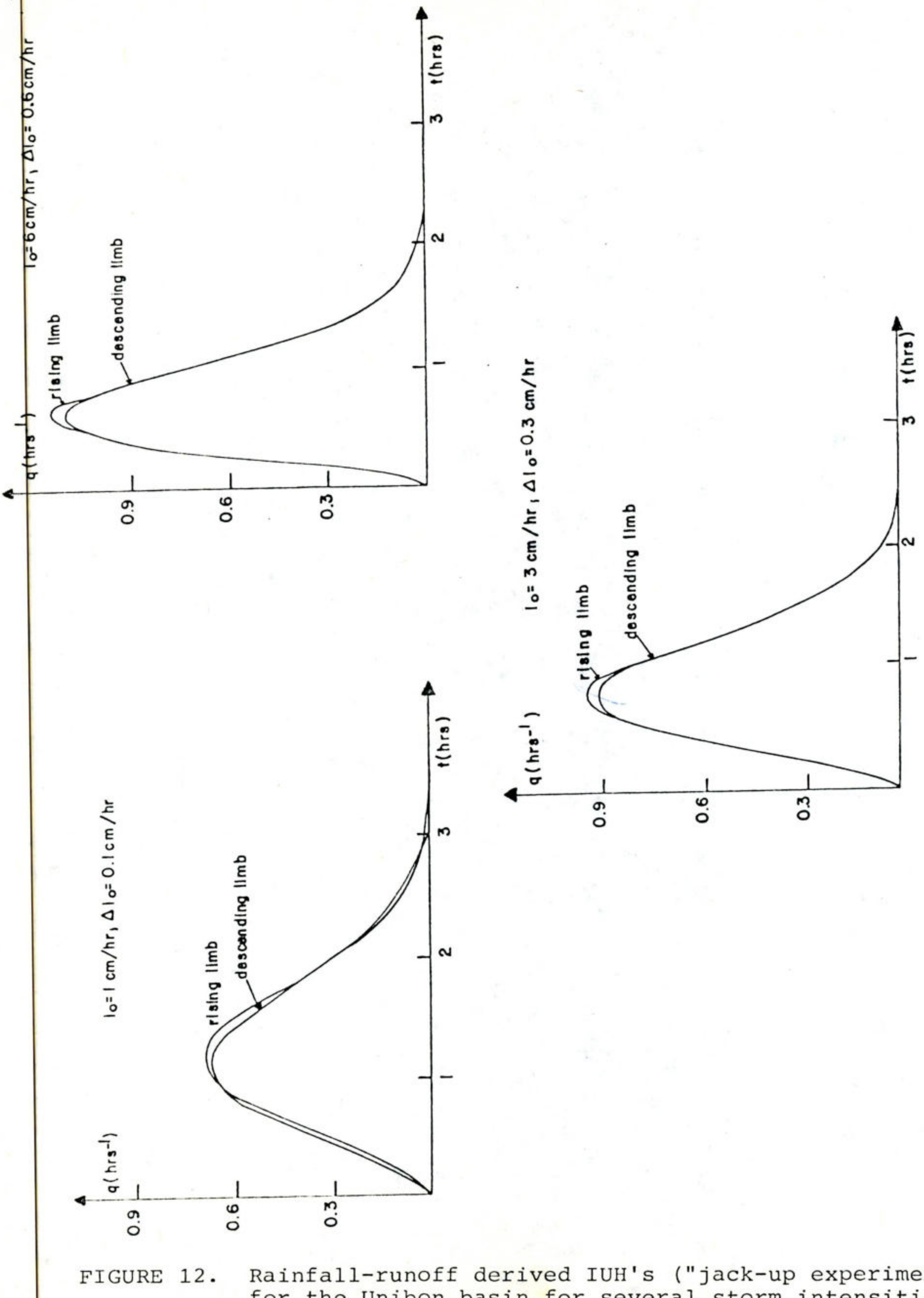


FIGURE 12. Rainfall-runoff derived IUH's ("jack-up experiments") for the Unibon basin for several storm intensities.

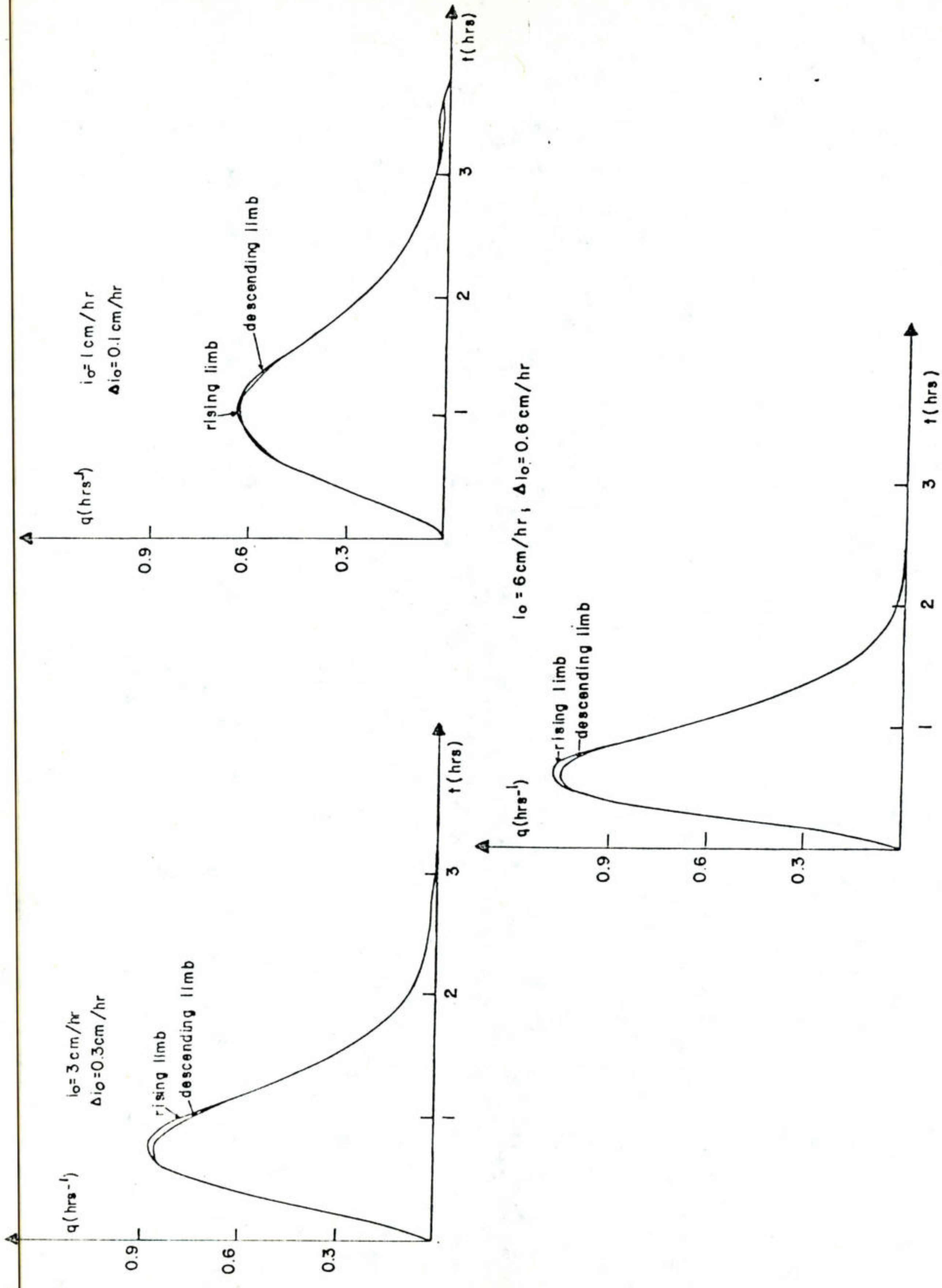


FIGURE 13. Rainfall-runoff derived IUH's ("jack-up experiments") for the Morovis basin for several storm intensities.

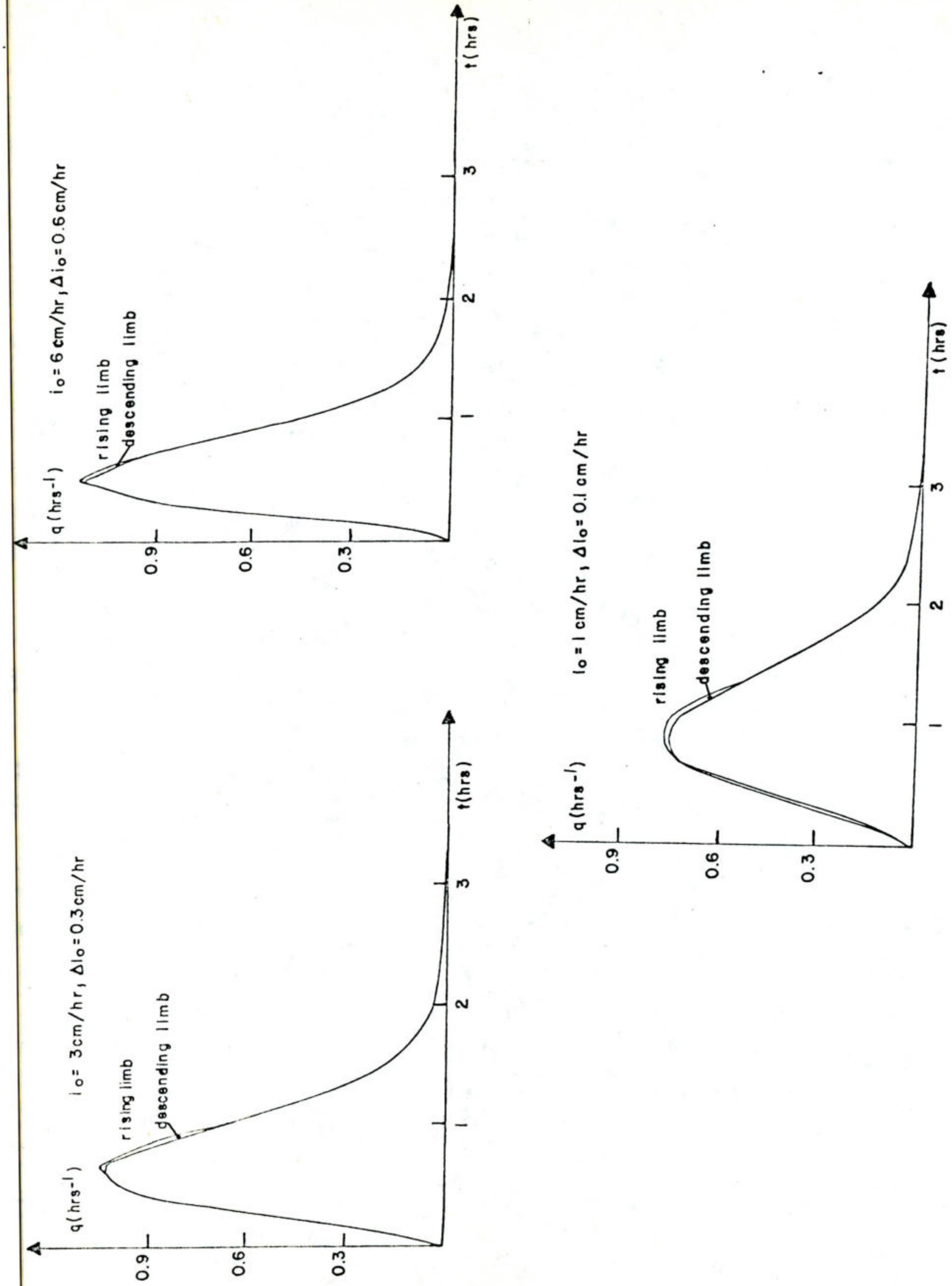


FIGURE 14. Rainfall-runoff derived IUH's ("jack-up experiments") for the Mamon basin for several storm intensities.

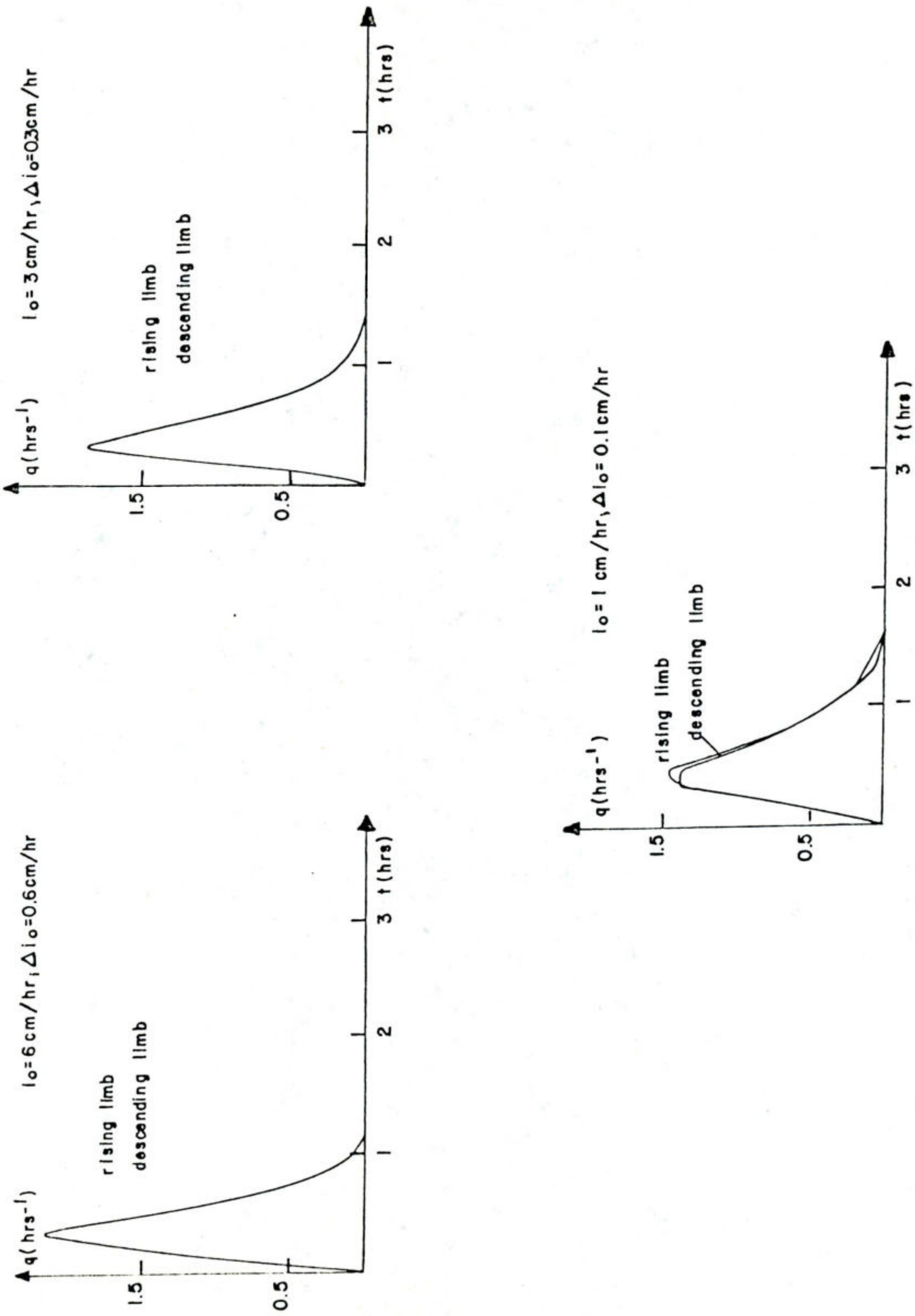


FIGURE 15. Rainfall-runoff derived IUH's ("jack-up experiments") for the Mamon 5 basin for several storm intensities.

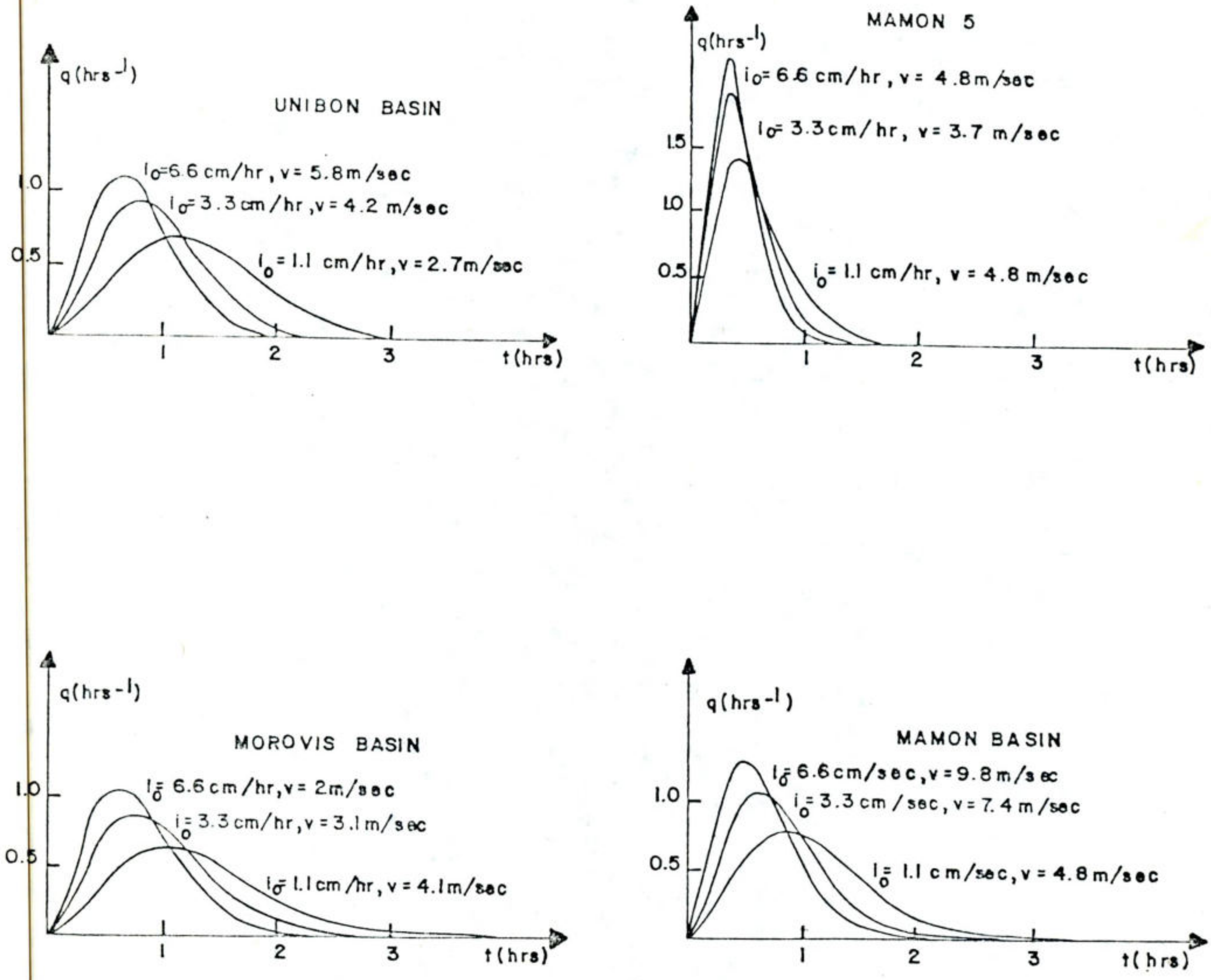


FIGURE 16. Variations in the response function of the four basins as a function of the storm intensities. ("Jack-up experiments").

constant velocity condition, this velocity is not the same for the different storms but is higher with the increase of the storm intensity.

The velocity at the time of equilibrium, $v_e$, given by the rainfall-runoff model, is now used to derive the geomorphologic IUH for each basin and for each storm intensity. The geomorphologic IUH's are then compared with the IUH's derived from the rainfall-runoff models using the "jack-up" experiments. The comparison is very satisfactory as shown in Figures 17, 18, 19 and 20. Thus the variation from storm to storm of the response function of the basin which are a non-linear function of storm intensities are satisfactorily handled by the dynamic component of the geomorphologic IUH. The variation within the storm of the response function which are also due to the non-linearities of the system could also be handled by a time-variant geomorphologic IUH in which v is changing with time. In the third paper (Rodríguez-Iturbe et al., 1978) the effect that these variations have on the discharge peak, $Q_p$ and time to peak discharge, $T_p$ are fully analyzed.

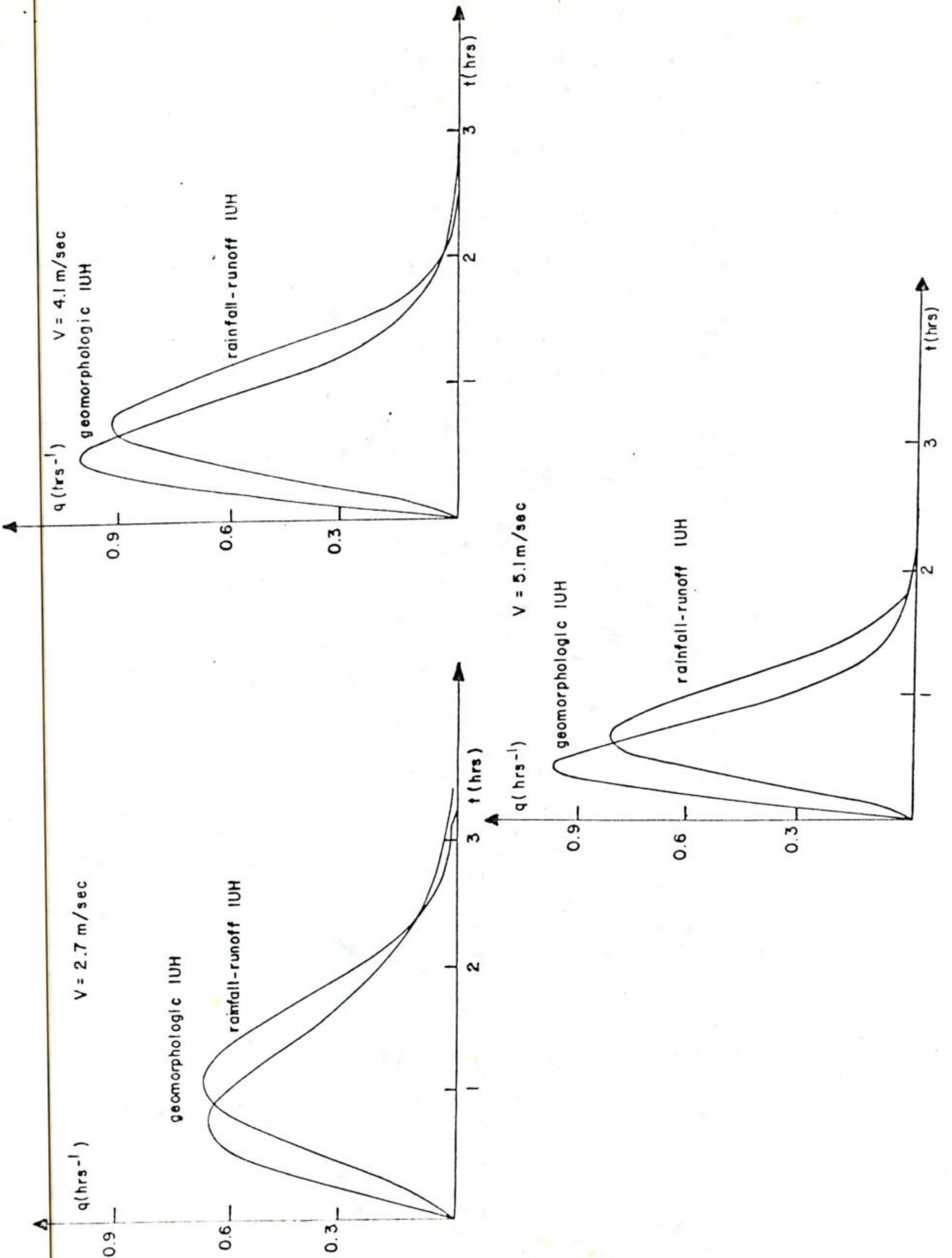


FIGURE 17. Comparison of the geomorphologic IUH's derived for the Unibon basin with rainfall-runoff derived IUH for different storm intensities.

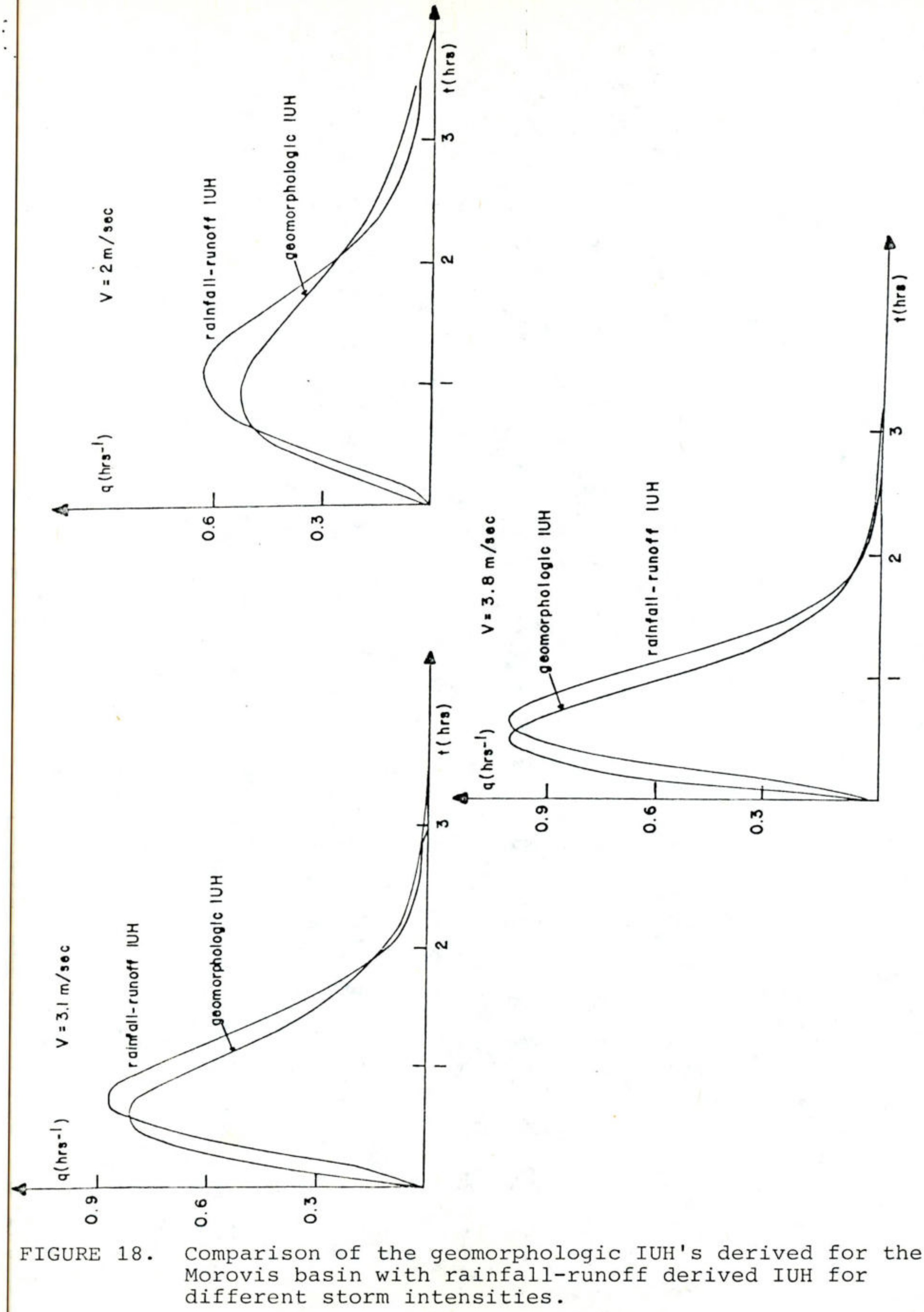


FIGURE 18. Comparison of the geomorphologic IUH's derived for the Morovis basin with rainfall-runoff derived IUH for different storm intensities.

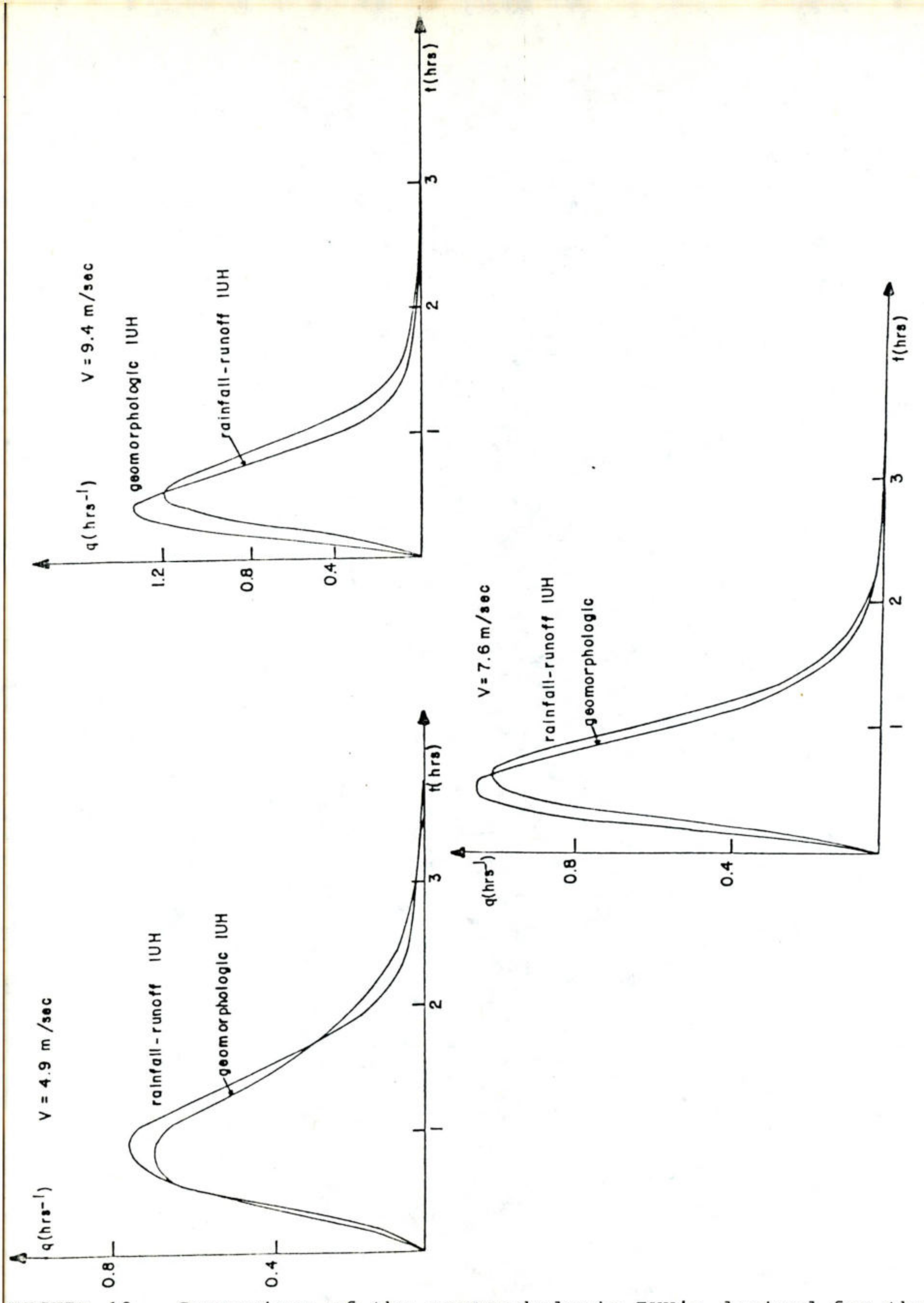


FIGURE 19. Comparison of the geomorphologic IUH's derived for the Mamon basin with rainfall-runoff derived IUH for different storm intensities.

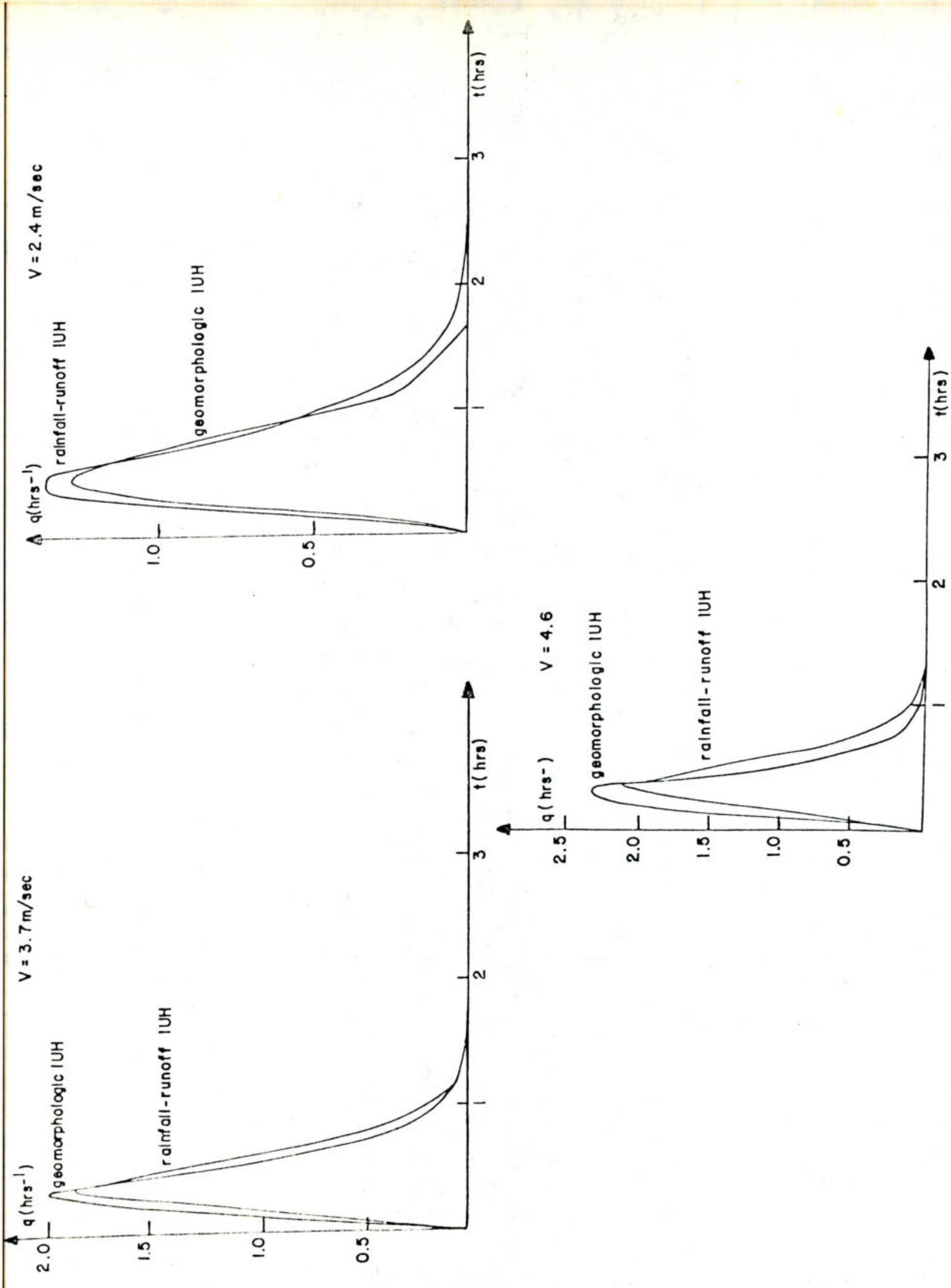


FIGURE 20. Comparison of the geomorphologic IUH's derived for the Mamon 5 basin with rainfall-runoff derived IUH for different storm intensities.

## 4. Summary and Conclusions.

The instantaneus unit hydrograph derived as a function of the geomorphologic parameters of a basin by Rodríguez-Iturbe and Valdés (1978) seems to be a workable approach to obtain the response function of a basin.

This is the result of controlled experiments which were carried out on four basins in Venezuela and Puerto Rico with areas ranging from 3 to 103 $km^2$ in which a geomorphologic analysis and a detailed rainfall-runoff modelling was made. The instantaneus unit hydrograph derived from the discharge hydrograph of the basins for different storm intensities were compared among themselves and with the geomorphologic IUH. No spatial or temporal variation of the precipitation was considered and the basins were assumed to be completely impervious since IUH theory deals with rainfall excess. As the experiments showed there is danger of serious errors when using IUH's derived from storms of different characteristics to those of the design storm. These non-linear characteristics of the response function of a basin can be modelled with a linear scheme such as the IUH but with a velocity that reflects the maximum discharge expected at the basin outlet. The variations of the velocity during the storm can also be incorporated with a true variant IUH throughout each storm event but this is not as important as the case of different storms with dissimilar rainfall intensities and durations.

# ACKNOWLEDGEMENTS

This research was carried out at Simón Bolívar University, Caracas, Venezuela. The finantial support of the Consejo Nacional de Investigaciones Científicas y Técnicas (CONICIT) of Venezuela through Grant S1-0759 is gratefully acknowledged. Special thanks are given to Dr. John Schaake for his suggestions in the experiments.

# REFERENCES

Resource Analysis Inc., "Hydrologic Impact of Coco Beach/Lagoon Project", Technical Report, Cambridge, Massachusetts, 1976.

Rodríguez-Iturbe, I., "Hydrologic Study of the Fajardo River Basin", Technical Report, Cambridge, Mass., 1974.

Rodríguez-Iturbe, I. and J. B. Valdés, "The Geomorphologic Structure of Basin Response", submitted for publication to Water Resources Research, September 1978.

Rodríguez-Iturbe, I., G. Devoto and J. B. Valdés, "Discharge Response Analysis and Hydrologic Similarity: The Interrelation between the Geomorphologic IUH and the Storm Characteristics", submitted for publication to Water Resources Research, September 1978.

Schaake, J.C., "Deterministic Urban Runoff Model", in "Treatise on Urban Water Systems", edited by M. L. Albertson, L. S.Tucker and D. C. Taylor, Colorado State University, 1971.

Schaake, J.C., "Rainfall-Runoff Modelling", Unpublished Course Notes, 1978.

# LIST OF FIGURES

Submitted Water Resources Research #3

# DISCHARGE RESPONSE ANALYSIS AND HYDROLOGIC SIMILARITY: THE INTERRELATION BETWEEN THE GEOMORPHOLOGIC IUH AND THE STORM CHARACTERISTICS

by

Ignacio Rodríguez-Iturbe, Gustavo Devoto and Juan B. Valdés

Graduate Program in Hydrology and Water Resources,
Universidad Simón Bolívar, Caracas, Venezuela.

# ABSTRACT

It is shown that the dynamic parameter v of the geomorphologic IUH can be taken as the velocity of the peak discharge time for a fixed rainfall-runoff event in a basin. This transforms the time variant IUH throughout the event in a time invariant IUH in each storm occurrence. The errors which the a priori estimation of the velocity in the IUH may cause in the calculation of the peak and time to peak of the runoff discharge are estimated for different types of basins and storms; the relative weights of the storm characteristics and the drainage network parameters in the prediction procedure are also studied in detail.

Drainage basins are defined as hydrologically similar if they have the same IUH when kinematic conditions are kept the same; through the geomorphologic IUH theory it is shown that the controlling parameter in hydrologic similarity is $R_L^{0.43} / L_\Omega$.

# INTRODUCTION

General expressions for the IUH have been derived as function of the geomorphologic parameters of a basin. The two most important parameters for a characterization of the IUH are its peak ($q_p$) and time to peak ($t_p$) which vary from storm to storm and also during a given storm as function of the velocity (v) of the flow at each time interval (Rodríguez-Iturbe and Valdés, 1978).

In this paper an attempt is made to investigate the problem of estimating the peak storm discharge ($Q_p$) and its time of ocurrence ($T_p$) from the geomorphologic IUH. Since the velocity of flow is unknown to the designer and moreover since this velocity changes during the life of the discharge response, two problem areas of interest are:

1) What kind of errors in $Q_p$ and $T_p$ can be expected from the uncertainty in the variable v?
2) What kind of errors in $Q_p$ and $T_p$ can be expected when assuming a constant value for the velocity of flow during a storm event? What kind of flow velocity should be used to characterize the different speeds of the drops at different time intervals during the storm?

To explore the above areas a fundamental tool is available in the rainfall-runoff models developed in hydrology in the last 15 years. Three of the basins described in the companion paper by Valdés et al. (1978) with the same very detailed rainfall-runoff representation have been used in this investigation to study the problem areas mentioned before.

The geomorphologic IUH derivation, clearly shows the two types of features of the response function: those depending only in the catchment characteristics and those which also depend on the rainfall intensity and duration and whose influence is reflected in the parameter v. The interrelation between these two types of features in combination with the rainfall intensity (i) and duration ($t_r$) produces the response discharge parameters $Q_p$ and $T_p$. The relative importance of the above type of features should shed some light in the nature of flood discharges occurring in natural watersheds and may also serve the further purpose of orienting the study of the much elusive and crucial problem of hydrologic similarity tackled later in this paper.

## FLOW VELOCITY CHARACTERIZATION

A storm of certain intensity constant throughout the duration $t_r$ is assumed to occur with an uniform spatial pattern over a watershed. Rodríguez-Iturbe and Valdés (1978) discuss in detail the assumption justified by several other investigators that the flow velocity at any given moment during the storm can be taken as more or less constant throughout the basin. The problem lies now in the fact that the above characteristic velocity for the basin as a whole, changes throughout time with the result that the IUH not only changes from storm to storm but also during the storm. Since using a time varying IUH which depends on a time varying velocity would greatly complicate any inferences about $Q_p$ and $T_p$ both for practical purposes and for theoretical generalizations, an attempt was made regarding the possibility of describing the time variant velocity with a single characteristic value.

Three of the basins described in Valdés et al. (1978), with different geomorphologic and physiographic characteristics, were tested for a range of storms of varying intensities and durations. By means of the rainfall-runoff characterization of each basin, histograms of the velocity distribution over the whole period of outflow were then obtained at the outlet of the basin. Figure 1 shows typical results of the analysis. In all cases the velocity distribution exhibits a relatively small variance in the sense that most of the flow is concentrated in a not too wide interval near the peak velocity. The variance of the distribution diminishes when increasing either the intensity of rainfall or its duration or both. This will tend to indicate that an attempt to use the geomorphologic IUH with a constant velocity equal to that at the peak discharge is a justifiable approach for the estimation of $Q_p$ and $T_p$.

The kinematic characteristics of the response process are then assumed to be synthetized in the maximum velocity expected during the outflow. We should now study the goodness of the above assumption. For this purpose the parameters of the IUH -$q_p$ and $t_p$- for the 3 basins in question, were estimated for a

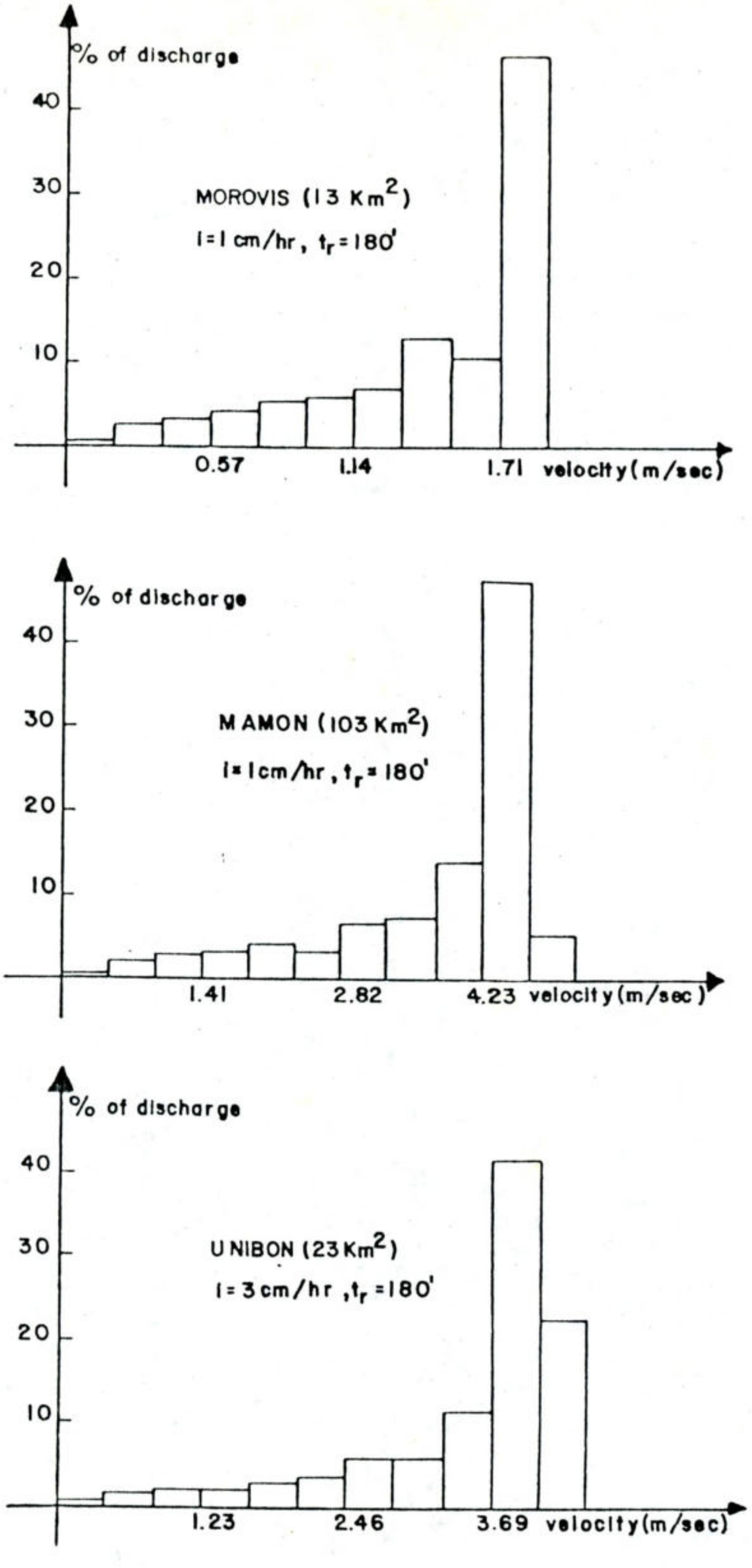


FIGURE 1. Examples of velocity histograms obtained by rainfall-runoff modelling at the outlet of different basins.

range of flow velocities going from 0.5 m/sec to 7 m/sec. To simplify the analysis a triangular form was assumed for the IUH since it is known that for prediction purposes the form itself is not important as long as the peak and time to peak of the IUH are correctly estimated (Henderson, 1963). Thus, a set of triangular IUH's was obtained for each basin, each IUH corresponding to a different flow velocity assumption.

Storms of different intensity and duration were then convoluted with each set of IUH's to obtain the $Q_p$ and $T_p$ predicted by the geomorphologic hydrograph. These $Q_p$ and $T_p$ were afterward compared with the equivalent values obtained when the storms are inputed into an impervious rainfall-runoff representation of each watershed.

Tables 1 through 3 give examples of the results obtained when comparing $Q_p^*$ and $T_p^*$ from the rainfall-runoff model with the $Q_p$ and $T_p$ obtained from the use of the geomorphologic IUH with a velocity equal to the velocity given in the rainfall runoff model at the moment of the peak discharge ($v_p^*$).

The comparisons are very encouraging suggesting that the geomorphologic IUH can indeed become an useful tool in many regions where very little or no data is available to the hydrologist. They also suggest that the approach can be an avenue to compare responses arising from different basins when different storms are involved.

Since $v_p^*$ is unknown to the hydrologist, the sensibility of $Q_p$ and $T_p$ to different assumptions on $v_p^*$ was studied estimating

Criterio
Horton ?
Strahler ?

Mamon: $R_A=4.5$, $R_B=3.5$, $R_L=2.1$, 6th order, $L_\Omega=12.25$Kms, $A=103\,\text{Km}^2$

---

| v(m/sec) | $T_p$(minutes) | $Q_p$($m^3$/sec) | |
|---|---|---|---|
| | i= 1 cm/hr, $t_r$= 3 hrs | | |
| 4.0 | 186 | 281 | $v_p^* = 4.6$ |
| 4.5 | 180 | 284 | $Q_p^* = 271$ |
| 5.0 | 164 | 285 | $T_p^* = 180$ |
| | i= 1 cm/hr, $t_r$= 2 hrs | | |
| 4.0 | 141 | 236 | $v_p^* = 4.4$ |
| 4.5 | 135 | 252 | $Q_p^* = 238$ |
| 5.0 | 130 | 265 | $T_p^* = 130$ |
| | i= 1 cm/hr, $t_r$= 1 hr | | |
| 3.0 | 113 | 112 | $v_p^* = 3.3$ |
| 3.5 | 103 | 128 | $Q_p^* = 113$ |
| 4.0 | 96 | 143 | $T_p^* = 110$ |
| | i= 1 cm/hr, $t_r$= 0.5hrs | | |
| 2.0 | 125 | 40 | $v_p^* = 2.2$ |
| 2.5 | 104 | 50 | $Q_p^* = 40$ |
| 3.0 | 91 | 59 | $T_p^* = 130$ |

TABLE 1. Examples of the comparisons for the Mamon basin of the peak discharges and time to peak obtained from the geomorphologic IUH with the equivalent values ($Q_p^*, T_p^*$) obtained by rainfall-runoff experiments.

Unibon: $R_A=5.6$, $R_B=4$, $R_L=2.8$, 3rd order, $L_\Omega=8.6$Kms, $A=23\text{Km}^2$

| v(m/sec) | $T_p$(minutes) | $Q_p$($m^3$/sec) | |
|---|---|---|---|
| | i= 3cm/hr, $t_r$= 3hrs | | |
| 2.5 | 185 | 192 | $v_p^* = 4.1$ |
| 3.0 | 170 | 194 | $Q_p^* = 194$ |
| 4.0 | 127 | 194 | $T_p^* = 180$ |
| | i= 3cm/hr, $t_r$= 2hrs | | |
| 3.5 | 126 | 188 | $v_p^* = 4.0$ |
| 4.0 | 122 | 193 | $Q_p^* = 188$ |
| 4.5 | 113 | 194 | $T_p^* = 120$ |
| | i= 3cm/hr, $t_r$= 1hr | | |
| 2.5 | 95 | 98 | $v_p^* = 3.2$ |
| 3.0 | 86 | 113 | $Q_p^* = 109$ |
| 3.5 | 81 | 127 | $T_p^* = 100$ |
| | i= 3cm/hr, $t_r$= 0.5hrs | | |
| 1.5 | 104 | 33 | $v_p^* = 2.2$ |
| 2.0 | 84 | 43 | $Q_p^* = 44$ |
| 2.5 | 72 | 53 | $T_p^* = 100$ |

TABLE 2. Examples of the comparisons for the Unibon basin of the peak discharges and time to peak obtained from the geomorphologic IUH with the equivalent values ($Q_p^*$, $T_p^*$) obtained by rainfall-runoff experiments.

Morovis: $R_A=5$, $R_B=3.2$, $R_L=2.7$, 3rd order, $L_\Omega=8$ Kms, A= 13$\text{Km}^2$

| v(m/sec) | $T_p$(minutes) | $Q_p$($m^3$/sec) | |
|---|---|---|---|
| | i= 3cm/hr, $t_r$= 3hrs | | |
| 2.5 | 182 | 112 | $v_p^* = 3.0$ |
| 3.0 | 159 | 112 | $Q_p^* = 112$ |
| 3.5 | 136 | 112 | $T_p^* = 180$ |
| | i= 3cm/hr, $t_r$= 2hrs | | |
| 2.5 | 136 | 97 | $v_p^* = 2.9$ |
| 3.0 | 129 | 105 | $Q_p^* = 103$ |
| 3.5 | 124 | 110 | $T_p^* = 130$ |
| | i= 3cm/hr, $t_r$= 1hr | | |
| 2.0 | 100 | 49 | $v_p^* = 2.3$ |
| 2.5 | 89 | 59 | $Q_p^* = 55$ |
| 3.0 | 82 | 68 | $T_p^* = 90$ |
| | i= 3cm/hr, $t_r$= 0.5hrs | | |
| 1.0 | 130 | 14 | $v_p^* = 1.5$ |
| 1.5 | 95 | 20 | $Q_p^* = 21$ |
| 2.0 | 77 | 26 | $T_p^* = 100$ |

TABLE 3. Examples of the comparisons for the Morovis basin of the peak discharges and time to peak obtained from the geomorphologic IUH with the equivalent values ($Q_p^*$, $T_p^*$) obtained by rainfall-runoff experiments.

the percentage errors $|Q_p - Q_p^*| / Q_p^*$ and $|T_p - T_p^*| / T_p^*$ for the same kind of experiment described before. Figures 2 through 4 show examples of the results obtained in this analysis. It is observed that when $v_p^*$ is above 2 m/sec one may err in the estimation of the velocity to be used in the geomorphologic IUH without producing serious errors in the estimation of the peak discharge and time to peak. Nevertheless in storms with smaller flow velocities, relatively small errors in the kinematic parameter of the IUH will lead to large errors in the estimation of $Q_p$ and $T_p$. For design considerations the above observations are somewhat tranquilizing since one is normally interested in the response of the basin to storms of critical nature which produce large flow velocities. The engineer could estimate $Q_p$ and $T_p$ for different velocity values in the IUH and make a decision type of analysis, accounting for the uncertainty in the velocity.

The above analysis also shows the danger of using unit hydrographs derived for storms of different character than those which the hydrologist wishes to study. Furthermore, it explains in an objective manner the different unit hydrographs that can be obtained in the same basin when performing the estimation in a routine type of numerical scheme applied to different rainfalls and their corresponding hydrographs.

## STORM AND DRAINAGE NETWORK CHARACTERISTICS: THEIR RELATIVE WEIGHTS

The assumption of a certain form for the IUH will lead to

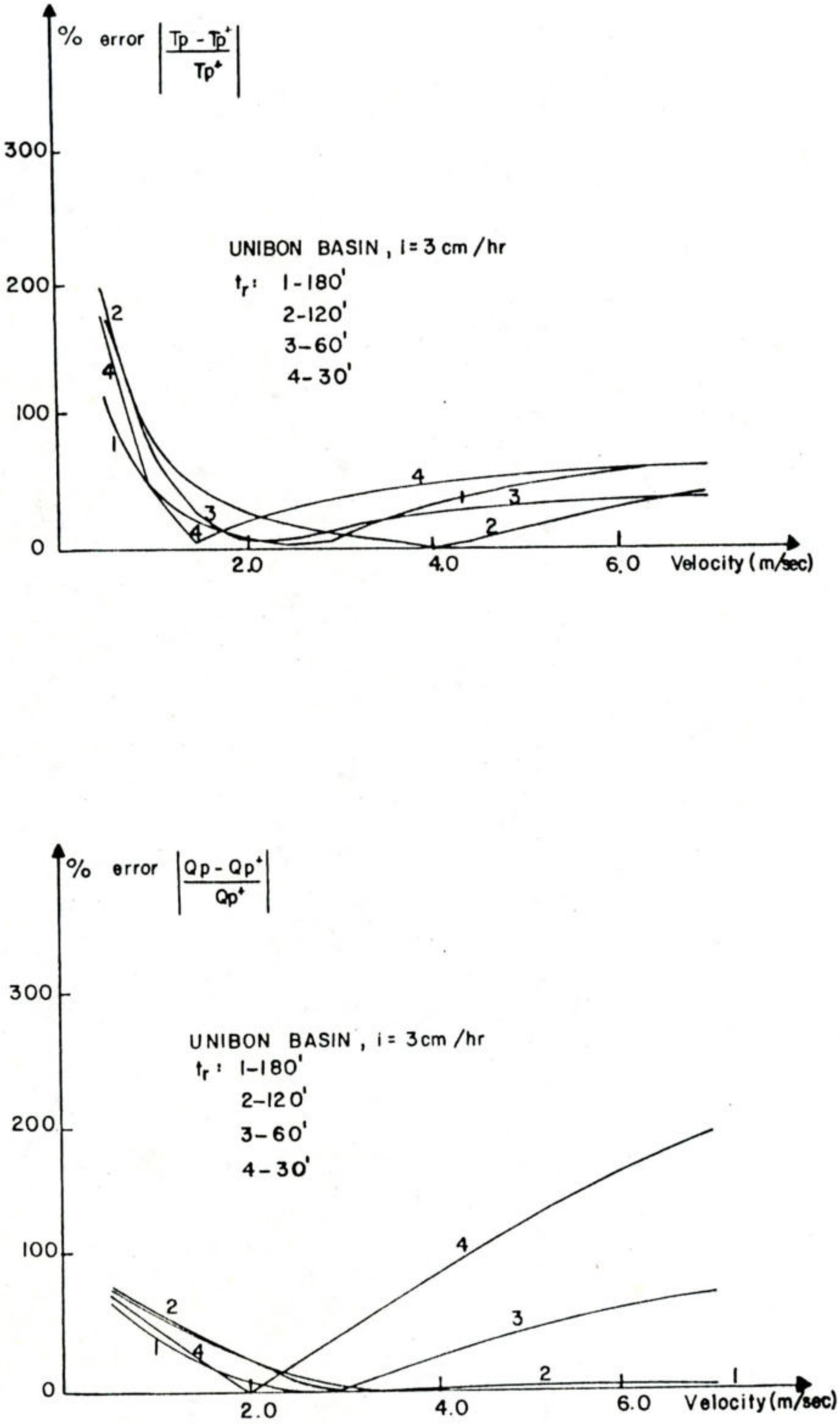


FIGURE 2. Examples of the errors made in the estimation of $Q_p$ and $T_p$ for several storms in the Unibon basin when different velocities are used in geomorphologic IUH.

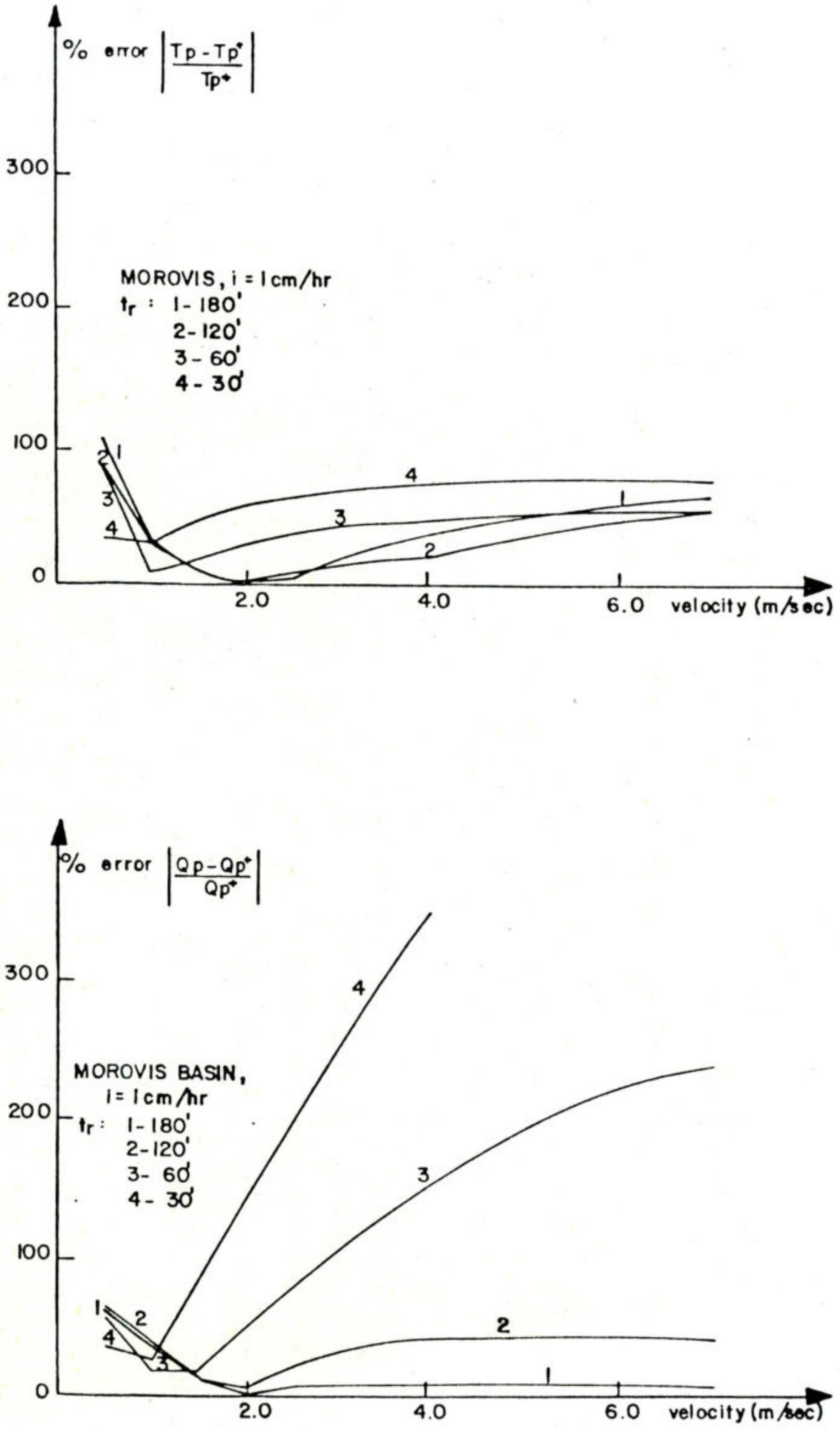


FIGURE 3. Examples of the errors made in the estimation of $Q_p$ and $T_p$ for several storms in the Morovis basin when different velocities are used in geomorphologic IUH.

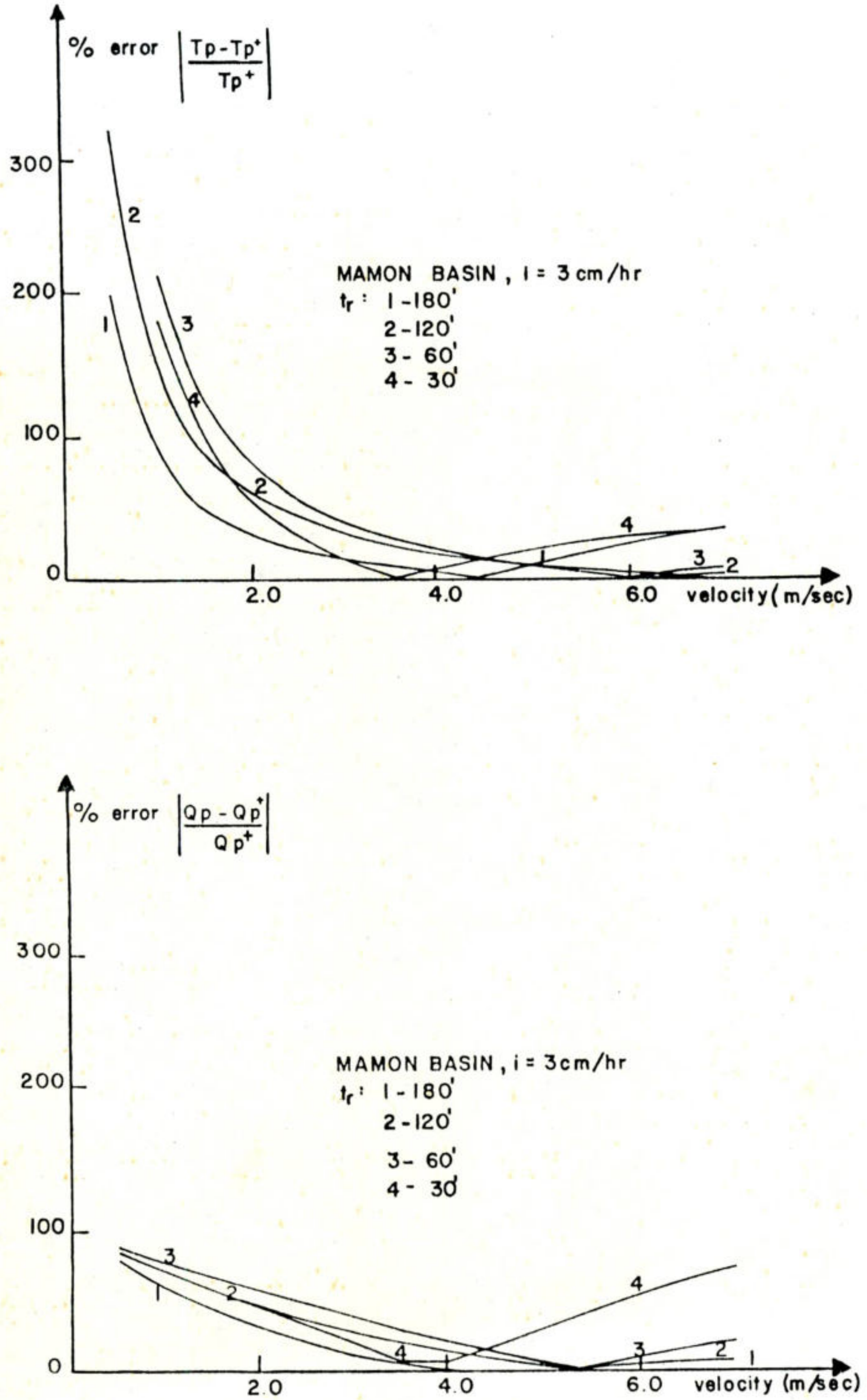


FIGURE 4. Examples of the errors made in the estimation of $Q_p$ and $T_p$ for several storms in the Mamon Basin when the different velocities are used in the geomorphologic IUH.

explicit relationships connecting the storm and drainage network characteristics in regard to their joint action to produce the peak outflow discharge. Henderson (1963) has shown that for an IUH of a triangular form the following relationship holds

$$\boxed{\frac{Q_p}{Q_e} = \frac{2t_r}{t_b} \left( 1 - \frac{t_r}{2t_b} \right)} \qquad (1)$$

where $Q_p$ is the peak discharge produced by a rainfall of constant intensity i and duration $t_r$, $Q_e$ is the equilibrium discharge iA (A, area of the basin) and $t_b$ is the base time of the IUH.

Since $q_p \times t_b = 2$, one may rewrite (1) as

$$\boxed{\frac{Q_p}{Q_e} = t_r \cdot q_p \left(1 - \frac{t_r \cdot q_p}{4}\right)} \qquad (2)$$

Using $\boxed{q_p = \frac{1.31}{L_\Omega} R_L^{0.43} v}$ (3) (Rodríguez-Iturbe and Valdés, 1978)

one may estimate the ratio $Q_p/Q_e$ for different combinations of $t_r$, v, $L_\Omega$, and $R_L$.

$L_\Omega$ - longitud del cauce que define el orden de la cuenca
$R_L$ - Radio de elongación $L_\Omega/L_{\Omega-1}$ ⇕

Figures 5 and 6 show examples of the above dependence. It is interesting to notice:

a) When v increases holding $t_r$ constant there exist a considerable increase of the ratio $Q_p/Q_e$. This means that storms of the same duration and intensity produce different peak responses depending on the flow rates in the drainage network previously to the storm.

b) Any correlation about peak responses of storms of different duration should take into account the flow

$t_r$ - duración de la tormenta
$t_b$ - tiempo base Hid. unitario
$Q_e$ - gasto de equilibrio = Int. lluvia x A.
$Q_p$ - Gasto pico Hid. unit.

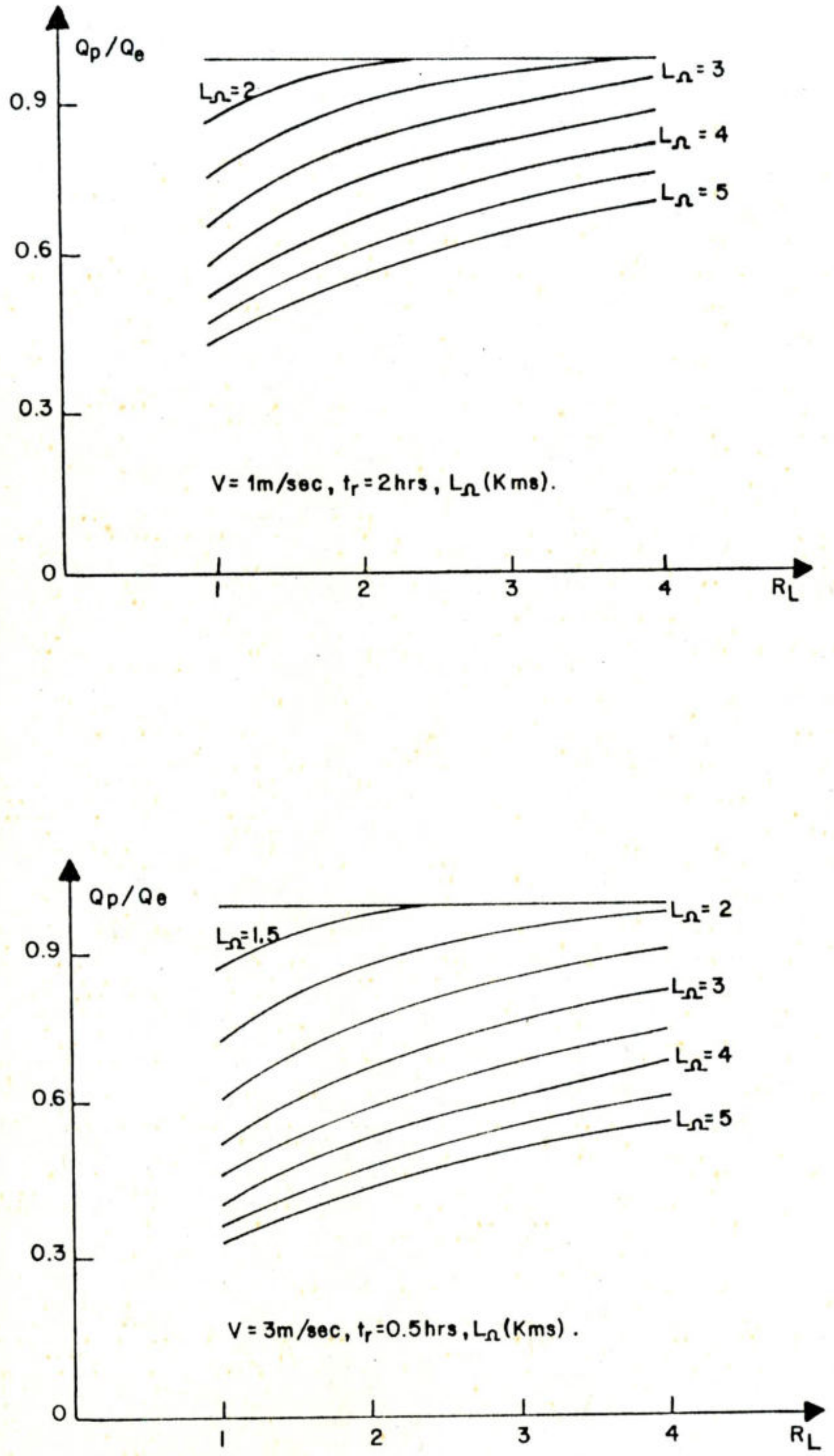


FIGURE 5. Relative weights of the storm characteristics and the drainage basin properties in the peak discharge from a rainfall event.

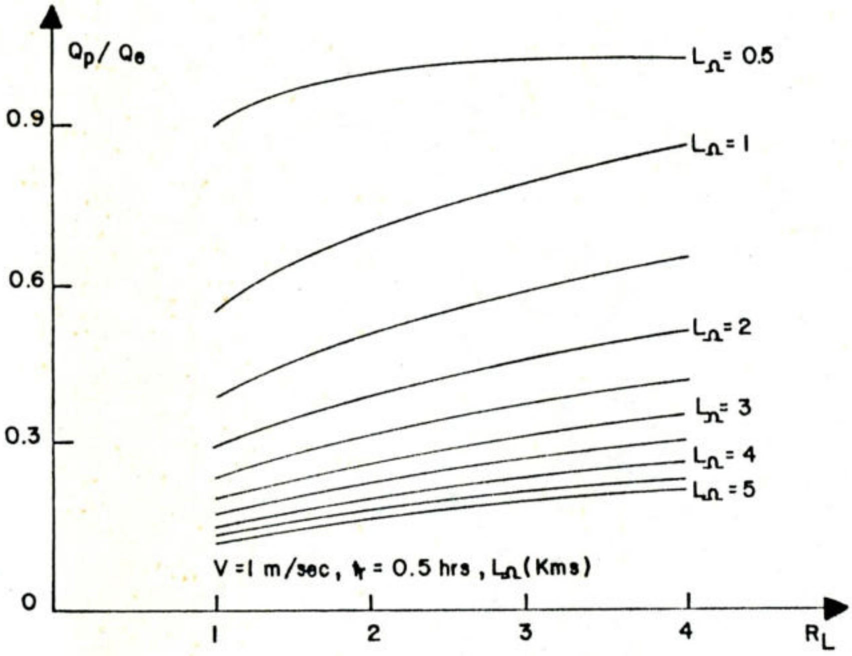


FIGURE 6. Relative weights of the storm characteristics and the drainage basin properties in the peak discharge from a rainfall event.

velocity parameter or the analysis will produce, either misleading results or lack of correlation.

c) The importance of the Horton's length ratio $R_L$ depends heavily on the characteristics of the storm and the internal scale parameter $L_\Omega$. Thus for the same external scale dimension-area- one observes that in all graphs it is below a certain value of $L_\Omega$ that $R_L$ starts playing an important role. For large values of $L_\Omega$ the relation $Q_p/Q_e$ changes very little through the whole range of $R_L$, this region is followed by a boundary after which $R_L$ seriously influences the ratio $Q_p/Q_e$. For the same duration of rainfall $(t_r)$ one observes that an increase in velocity will produce lines of the same $L_\Omega$ that were not being influenced by $R_L$ to move up to the region where the peak discharges are not only higher but also being dependent on $R_L$. The same observation is valid when the velocity is held constant but the rainfall duration increases.

d) The internal scale parameter $L_\Omega$ is a crucial factor in the variability of the ratio $Q_p/Q_e$.

## HYDROLOGIC SIMILARITY

Two basins will be defined as similar from a hydrologic point of view if when excited by an unit impulse of rainfall their response is the same given that the kinematic conditions are kept the same in both cases. Two comments should be made regarding the above definition, the first one concerns the fact we have defined hydrologic similarity through the IUH and not through comparisons of forms, areas, slopes, etc. We feel the most representative feature of the behavior of a system is its response function and any attempt addressed to compare the discharge-behavior of different basins through a general framework, should be based in the structure of an IUH which does not include parameters to be obtained through calibration. The second comment concerns the fact that kinematic conditions are kept the same in both basins. The theory of the geomorphologic IUH (Rodríguez-Iturbe and Valdés, 1978) and the experiments presented in Valdés et al. (1978), show the flow velocity v as the kinematic parameter which controls both the peak ($q_p$) and time to peak ($t_p$) of the IUH. Thus, the IUH depends besides of the input itself in the antecedent conditions of the basin, namely in the flow conditions in the drainage network at the moment we impose the unit impulse of rainfall. Since these conditions are independent of the event under study -the response to an unit input- we are forced to stablish they are identical in order to try a general study of the problem.

Our first goal in this section is the design of two basins which although different in many parameters and specially in the appearance of their drainage network will be hydrologically similar according to the theory of the geomorphologic IUH. Since we wish identical $q_p$ and $t_p$ we will need the same $\mathbb{R} = q_p . t_p$ which

Rodríguez-Iturbe and Valdés (1978) have shown to be independent of velocity and size and equal to:

$$\mathbb{R} = 0.58 \left( \frac{R_B}{R_A} \right)^{0.55} \qquad (4)$$

$\mathbb{R}$ was chosen the same for the two basins and then two different $R_A$'s were picked, through equation (4) the corresponding $R_B$'s were established. Assuming $\Omega = 3$ for both basins, $N_1$ and $N_2$ are obtained. In this experiment the same area is given to both basins (100 $Km^2$) and through $R_A$, we calculate $\overline{A}_1$ and $\overline{A}_2$. Regarding the choice of the $\overline{L}_\omega$'s we proceeded in the following manner: $\overline{L}_1$ was chosen through the functional form

$$\overline{L}_1 = \eta \overline{A}_1^{\ 1/\beta} \qquad (5)$$

For this, 9 different basins were analyzed to obtain values of $\eta$ and $\beta$ with some resemblance to reality. The only purpose of equation 5 is to avoid a choice of $\overline{L}_1$ totally independent of $\overline{A}_1$ which has already been chosen. Different $R_L$'s were used but in such a manner that the ratio $R_L^{0.43}/L_\Omega$ in the expression of $q_p$ remains the same in both basin.

The channels of order 1 draining to order 2 and 3 were distributed according, approximately, to the expressions of $p_{12}$ and $p_{13}$ given by Rodríguez-Iturbe and Valdés (1978). For the areas draining directly to each order-$\omega$ stream the vector $\theta_i(o)$ of initial state probabilities was used. Table 4 shows a summary of the characteristics of the two basins and figures 7 and 8 give the schematic of their drainage network.

The individual lengths of the streams of orders 1 and 2

| | BASIN 1 | BASIN 2 |
|---|---|---|
| $\Omega$ | 3 | 3 |
| G | .75 | .75 |
| $R_A$ | 4 | 6 |
| $R_B$ | 3 | 4.5 |
| $N_1$ | 9 | 21 |
| $N_2$ | 3 | 5 |
| $A=A_3$ | 100 $Km^2$ | 100 $Km^2$ |
| $\bar{A}_1$ | 6.25 $Km^2$ | 2.77 $Km^2$ |
| $\bar{A}_2$ | 25 $Km^2$ | 16.66 $Km^2$ |
| $\bar{L}_1$ | 4.58 Km | 2.87 Km |
| $R_L$ | 1.5 | 2 |
| $\bar{L}_2$ | 6.88 Km | 5.73 Km |
| $\bar{L}_3$ | 10.32 Km | 11.46 Km |
| $\theta_1(o)$ | .5625 | .5625 |
| $\theta_2(o)$ | .2625 | .3242 |
| $\theta_3(o)$ | .1750 | .1143 |
| $p_{12}$ | .8667 | .7569 |
| $p_{13}$ | .1333 | .2431 |

TABLE 4: Geomorphological characteristics of Basins 1 and 2 used in the experiments of hydrologic similarity.

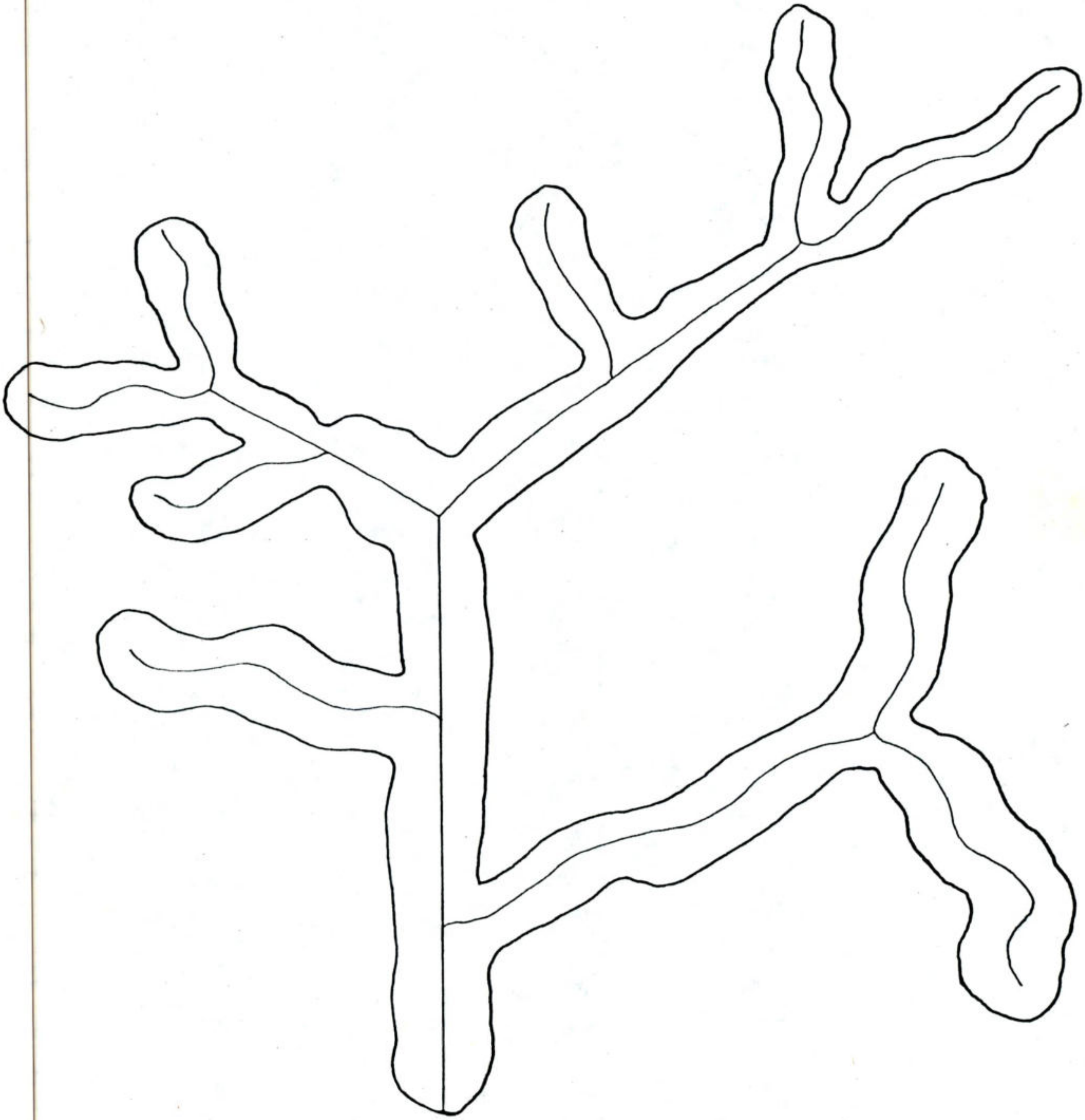


FIGURE 7: Schematic of Basin 1 used in the experiments of hydrologic similarity.

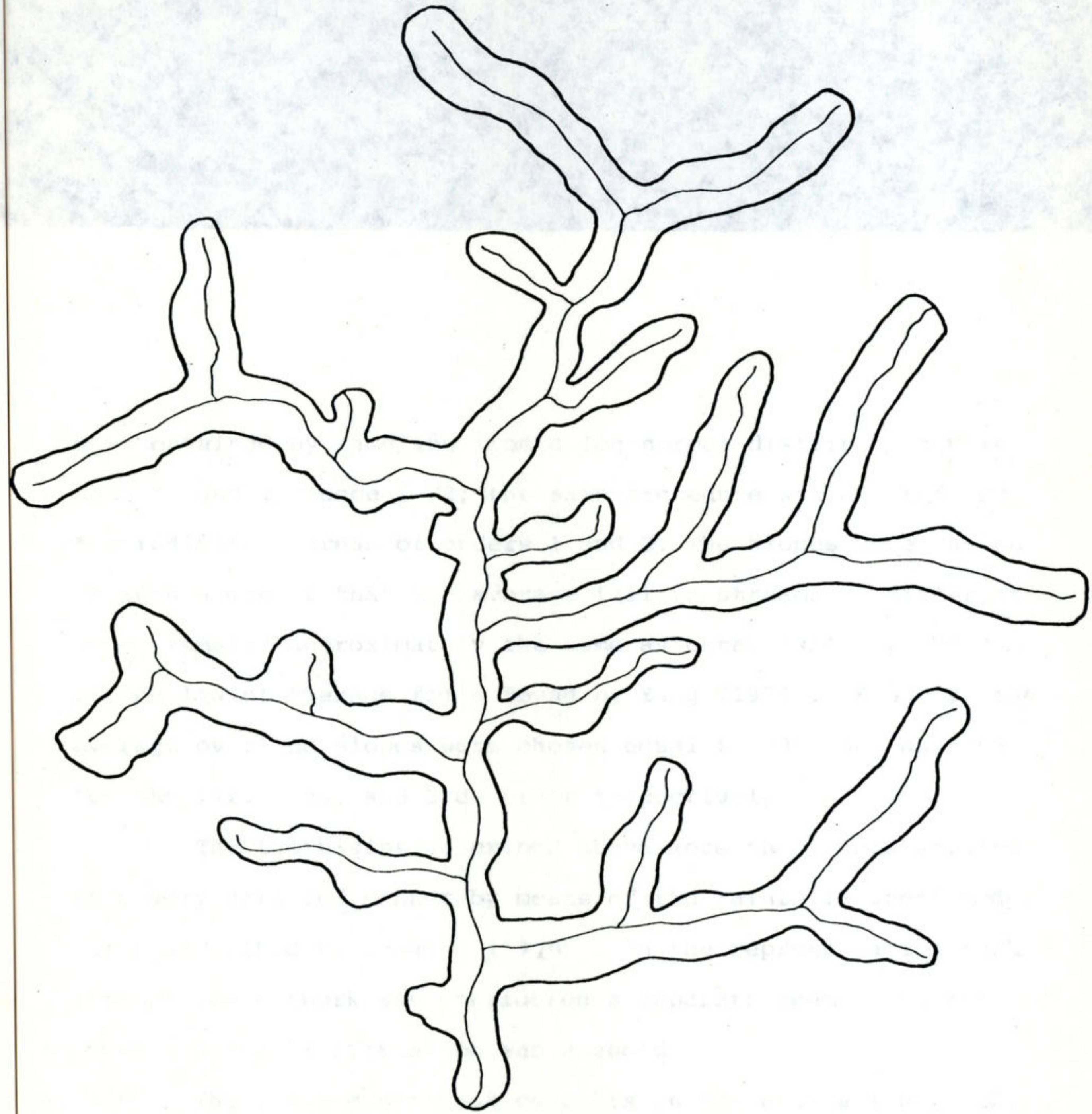


FIGURE 8. Schematic of Basin 2 used in the experiments of hydrologic similarity.

fall intensity. The kinematic conditions in both basins are kept the same because having the same total area, and the same average width of channel, at equilibrium time, they have the same flow velocity.

Figure 10 show typical examples of the IUH obtained in this experiment; it is observed the agreement is very good in

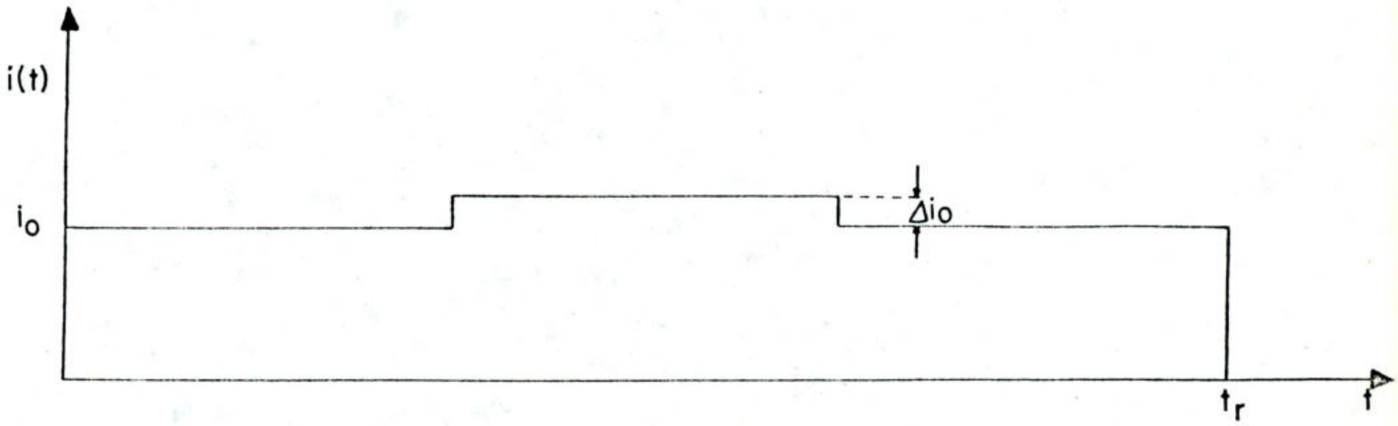


FIGURE 9. General hyetograph imposed upon the rainfall-runoff model to obtain the IUH to be compared with the geomorphologic theory ($\Delta i_0 = 0.10\ i_0$)

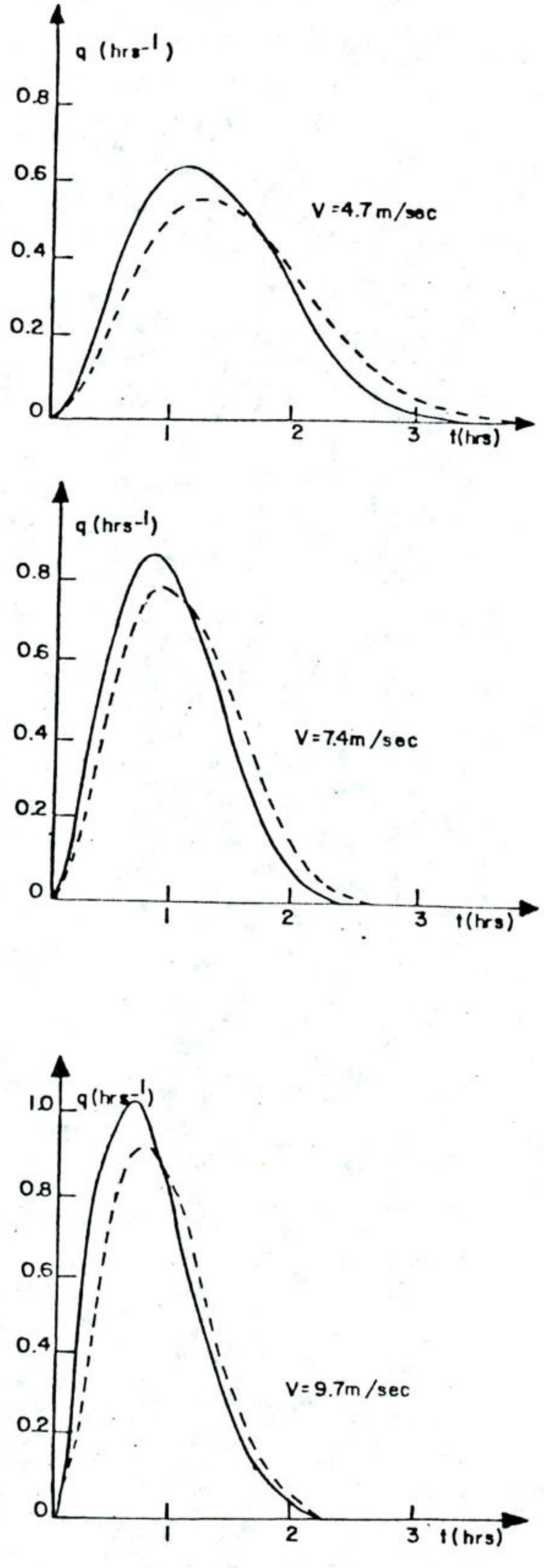


FIGURE 10. IUH's of Basin 1 (------) and 2 (———) for different kinematics conditions.

all cases. This we feel is not something that could be expected before-hand; one basin has 51 segments and the other 123 segments in the rainfall-runoff representation, the different basins of order 3 and of the same external scale-area- which could be constructed fulfilling Horton's laws are practically infinite and nevertheless two particular networks were built-up which although looking quite different to the naked eye, were predicted to have the same IUH.

The second experiment consisted in the comparison of two pairs of basins where the scale of lengths is 1:2. The two watersheds designed in the first experiment have areas of 100 $Km^2$, in each of them all longitudes of streams were multiplied first by a factor of 3 in order to have two basins of 300 $Km^2$. This nevertheless does not change the parameters $R_B$, $R_L$, $R_A$, $P_{ij}$, $\theta_i(o)$ which as well as the connectivity of the network remain the same as before. What has changed is the channel maintenance constant or drainage area per unit length of channel, nevertheless we do not worry about this because this parameter (or equivalently the drainage density which is its inverse) has a very wide range of variation in nature. The factor of 3 was applied in order not to have very small basins when the scale of lengths is reduced by 2 leading to a reduction by 4 in the area. Thus, we now have two pairs of basins, one pair of 300 $Km^2$ and the other pair of 75 $Km^2$. We will compare each one of the larger ones with its corresponding reduction in scale 1:2. In order to have the same kinematic conditions the rainfall-runoff representation of the larger basins was subjected to an intensity of rainfall 2 times larger than that of the smaller basins in each of the runs carried out. To avoid very large and unrealistic velocities the experiments are now carried out with rainfall intensities ranging from 0.2 cm/hr to 1.0 cm/hr and as before after reaching equilibrium a jack-up in the rainfall intensity is introduced equal to 10% of the base intensity.

Figure 11 is typical of the results of the experiment. The theory of the geomorphologic IUH indicates that in the corresponding pairs of basins the peak of the smaller one should be double of the larger one and the time to peak one half due to the fact that they have the same $R_L$, the same $R_A/R_B$ and the length $L_\Omega$ has been reduced by a factor of 2. The results from the rainfall-runoff model confirm extremely well this prediction.

One important point to bring up at this moment is the observation that for the same kinematic conditions, the effect of size or scale in the IUH does not come through the area of the basin but rather through the length of the streams reflected in the parameter $L_\Omega$ . Neither the expression of $q_p$ nor that of $t_p$ contain the area, they involve $R_L$, $R_A/R_B$ and $L_\Omega$ in addition to the flow velocity v. Moreover they are general expressions for watersheds of any order $\Omega$. This will suggest that basins of very different areas, number of sources, orders, etc. may indeed have the same IUH as long as

$R_L^{0.43}/L_\Omega$ which controls $q_p$

and

$L_\Omega \cdot (R_B/R_A)^{0.55} R_L^{-0.38}$ which controls $t_p$

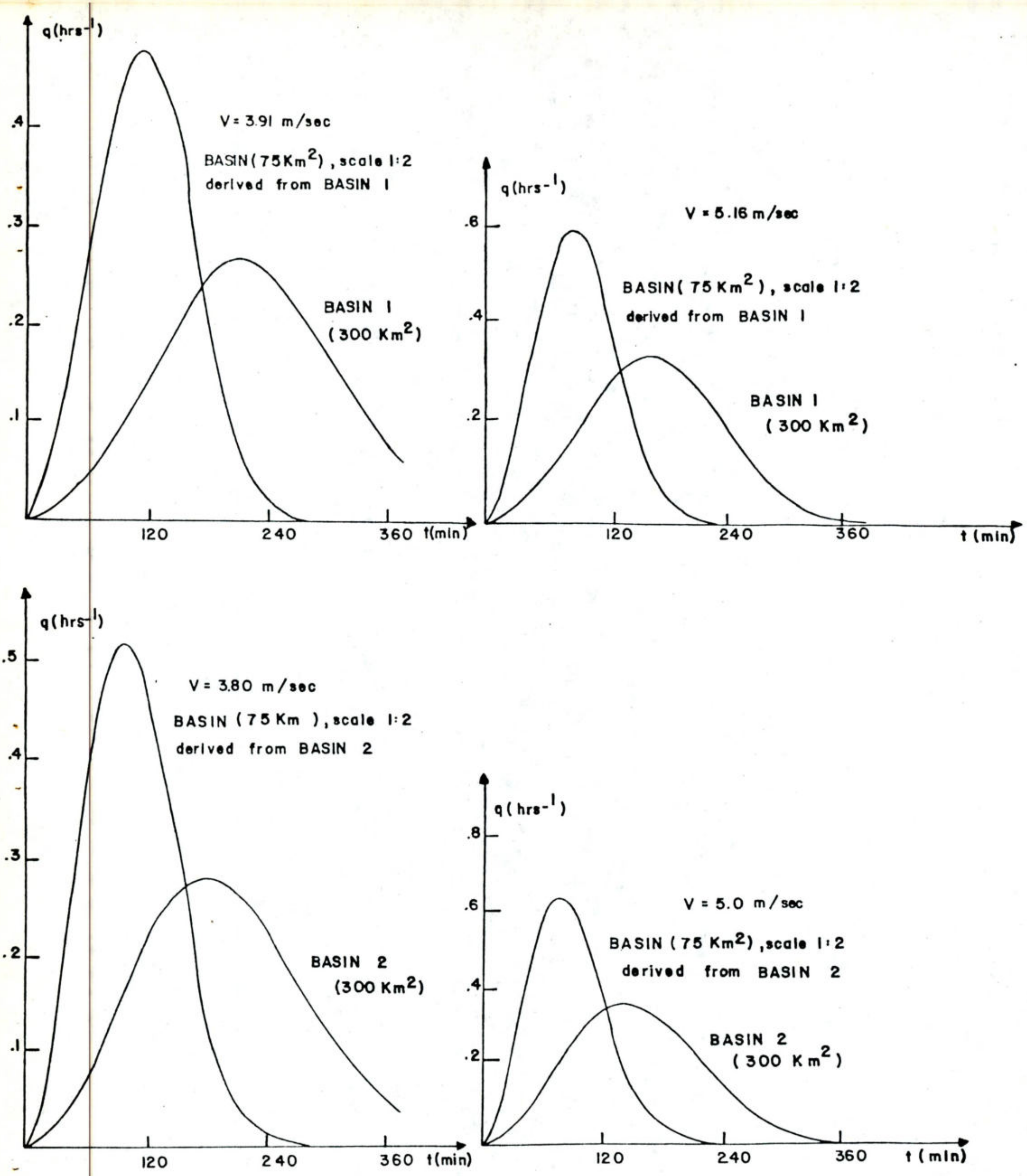


FIGURE 11. Examples of the IUH's obtained in the second experiment of hydrologic similarity. The theory suggest that the peak and time to peaks of the corresponding pais of hydrographs should be in a 1:2 relationship.

remain the same.

A third experiment was designed to verify the above observation. We started with Basin N°1 used in the first experiment -shown in Figure 7- and made this basin grow to a fourth order basin in the following manner: $R_B$, $R_L$, $R_A$ and $L_\Omega$ remain the same as in Basin N°1. Since the new basin is of order 4 we have 27 first order streams with average length of 3.06 Km. and $\overline{A}_1=2.7\,\mathrm{Km}^2$. The transition probabilities for a 4th order basin are given by

$$p_{12} = \frac{2}{R_B} + \frac{(2R_B-1)(R_B^2-2R_B)}{R_B^2\ (2R_B-1) + R_B\ (R_B^2-1) + (R_B^2-1)\ (R_B-1)} \tag{6}$$

$$p_{13} = \frac{(R_B^2-1)\ (R_B-1)}{R_B^2(2R_B-1) + R_B(R_B^2-1) + (R_B^2-1)\ (R_B-1)} \tag{7}$$

$$p_{14} = \frac{(R_B^2-1)\ (R_B-1)\ (R_B-2)}{R_B^3(2R_B-1) + R_B^2(R_B^2-1) + R_B(R_B^2-1)\ (R_B-1)} \tag{8}$$

$$p_{23} = \frac{R_B-2}{2R_B-1} + \frac{2R_B}{R_B^2} \tag{9}$$

$$p_{24} = \frac{(R_B - 1)}{R_B(2R_B-1)} \cdot (R_B - 2) \tag{10}$$

(Rodríguez-Iturbe et al,1978)

The streams of order 1 and 2 were distributed in Basin 3 according aproximately to the above expressions and the lengths and areas of the individual streams were generated in the same manner as in construction of Basins 1 and 2. Figure 12 shows

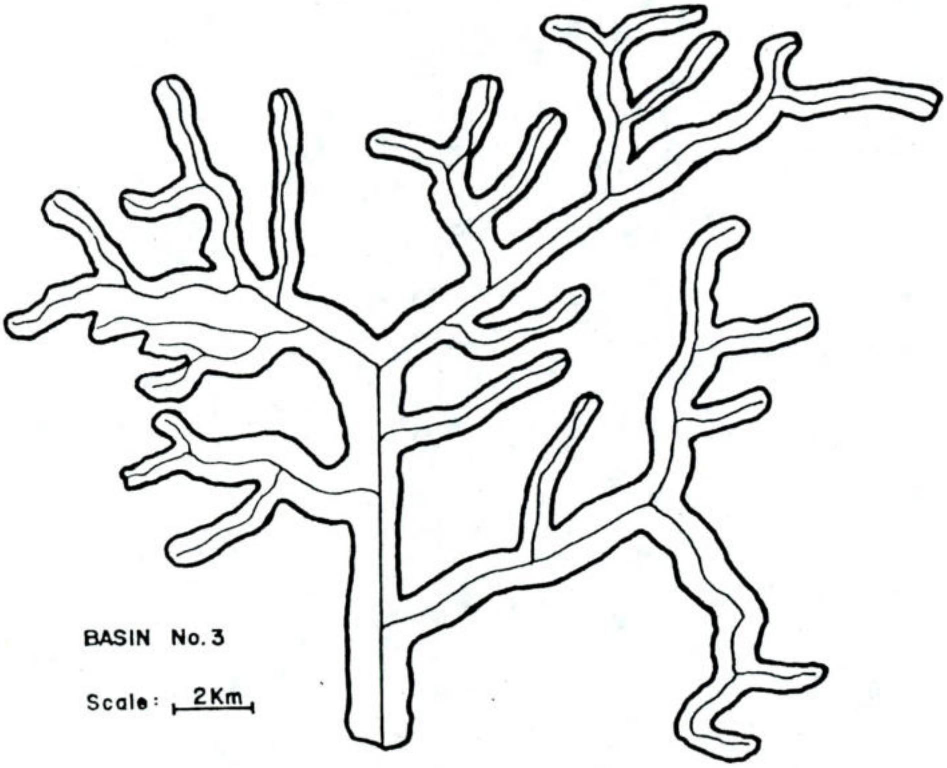


FIGURE 12. Schematic of Basin 3 used in the third experiment of hydrologic similarity.

Basin 3 which has a total area of 173 $\mathrm{Km}^2$ almost twice as Basin 1. A simple inspection of both basins, 1 and 3, show how different they are in this characteristics and appearance; nevertheless our prediction is that they should have the same IUH. Figure 13 shows an example of the results obtained from the rainfall-runoff model, in this case the same kinematic conditions were maintained through the use of different rainfall intensities in both basins. The agreement is again quite satisfactory.

The above experiments suggest that for the same kinematic conditions two basins may be considered hydrologically similar when they have identicals

$$R_L^{0.43}/L_\Omega \quad \text{and} \quad L_\Omega \cdot (R_B/R_A)^{0.55}\, R_L^{-0.38}$$

Since for the values of $R_L$ encountered in nature we may assume that $R_L^{0.43} \simeq R_L^{0.38}$, two basins will be similar when they have equal values of

$$(R_L^{0.43}/L_\Omega) \quad \text{and} \quad (R_B/R_A)$$

where $L_\Omega$ should be expressed in Kms when comparing different values of $(R_L^{0.43}/L_\Omega)$,(Rodríguez-Iturbe and Valdés, 1978). The relative weights of $R_L$ and $L_\Omega$ in the discharge response of a watershed were studied before in this paper.

The role of $(R_B/R_A)$ can be focused from a different perspective. The parameter $\mathbb{R}$ -equation 4- is a constant independent of the internal scale $L_\Omega$ and the kinematic condition v. For a triangular IUH $\mathbb{R}$ represents twice the area of the triangle defined by the origin, the abcissa $t_p$ and the ordinate $q_p$. We may think

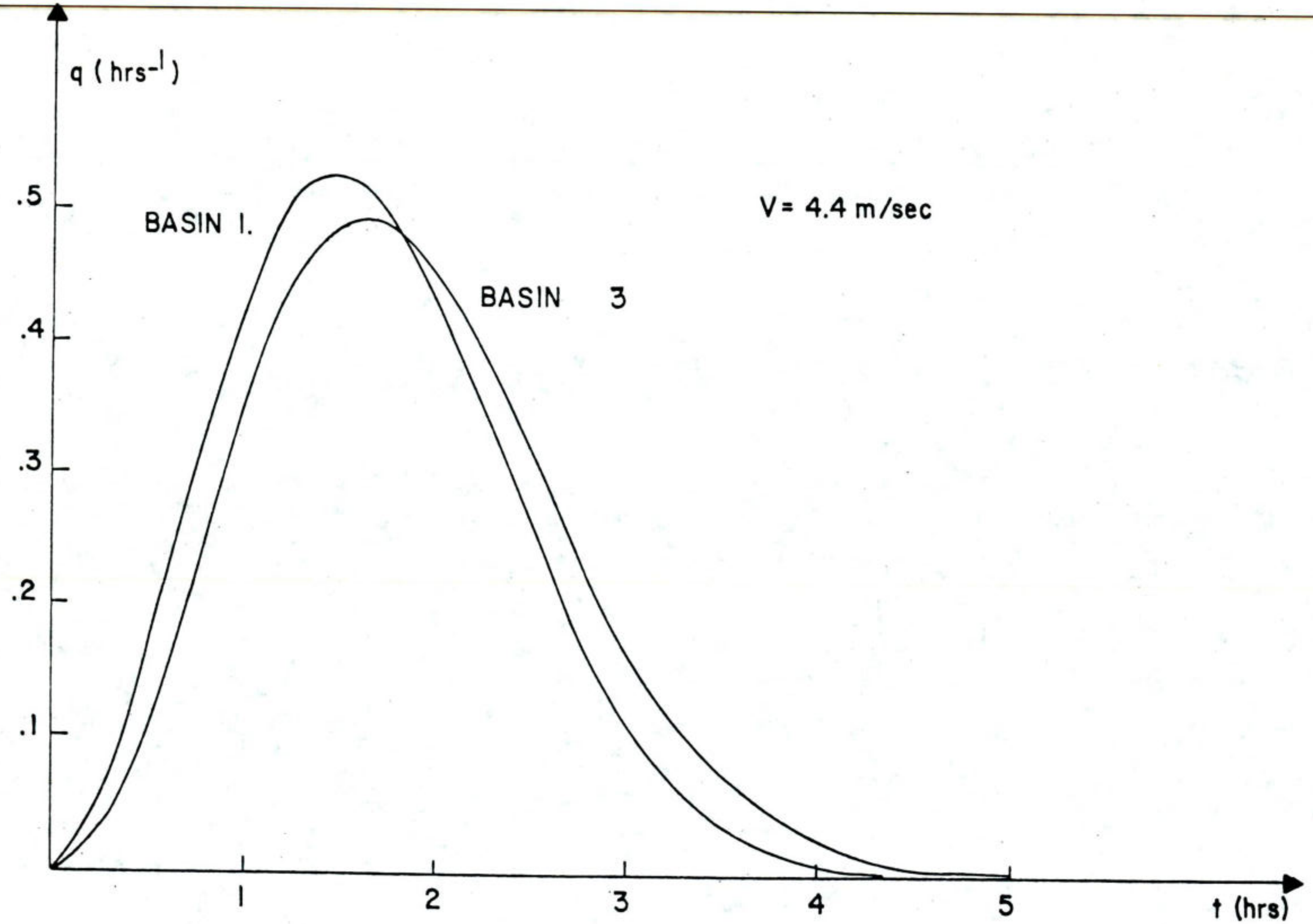


FIGURE 13. IUH's of Basins 1 and 3 which the theory suggest should have the same response function.

for a moment that our IUH is a two state system where a drop is found at the outlet either before $t_p$ or after $t_p$. Thus, the most probable shape of the IUH would be an isosceles triangle where $t_p = 0.5t_b$ and $\mathbb{R} = 1$. With $\mathbb{R} = 1$ the entropy of the probability function represented in the IUH will be a maximum and thus the most probable value of $R_B/R_A$ would be 2.68. Nevertheless, that would be a maximization of entropy without constraints and it is wrong since there are physical restrictions in the relative values $R_A$ and $R_B$ can take in a basin which follows Horton's geomorphological laws.

To start with, for a basin of any order $\Omega$ the probability that a drop lands in area of order 1 is given by

$$\theta_1(o) = \frac{N_1\bar{A}_1}{A_\Omega} = \left(R_B/R_A\right)^{\Omega-1} \tag{11}$$

and thus $R_A$ have to be larger or equal to $R_B$. Moreover if $R_A = R_B$, $\theta_1(o) = 1$ and then $\theta_i(o)$ for $i > 1$ should be equal to zero. Thus $R_A$ should be larger than $R_B$, we have checked in the literature and this was always the case except in some cases where the estimation of $R_B$ was improperly done (e.g., the point $N_\Omega = 1$ was not taken into account and the line $\log N_\omega$ -vs- $\log \omega$ is improperly drawn). The expressions for $\theta_2(o)$, $\theta_3(o)$, etc., impose further restrictions in the values of $R_B/R_A$ for those probabilities to be smaller than 1. A calculation was run by Rodríguez-Iturbe and Valdés (1978) for basins of order 3, 4 and 5 computing the values of the $\theta_i(o)$ for many values of $R_B$ and $R_A$. They found that the $\theta_i(o)$ were positive for $R_B/R_A \leq 0.80$.

Since the entropy of the IUH is a maximum for the largest

$R_B/R_A$ we will suggest that in nature the most probable value of this ratio is around 0.80 and its range of variation should be quite narrow since the number of particles (drops) involved in the IUH is very large making the most probable state almost a certain one for practical purposes.

Figure 14 shows a plot of the values of $R_B/R_A$ found in Morisawa (1962), Woodyer and Brookfield (1966) and Valdés et al. (1978). The ordinate is the entropy H

$$H = -\Sigma p \cdot \log p = -\mathbb{R}/2 \log \mathbb{R}/2 - (1 - \mathbb{R}/2) \log (1 - \mathbb{R}/2) \tag{12}$$

It is observed that most of the values are between 0.7 and 0.94. Although of course more data should be analyzed and moreover the estimation of $R_A$ and $R_B$ should be made under very objective and uniform criteria, the results suggest that indeed $R_B/R_A$ is pretty constant in nature.

The work of Shreve (1966) and more recently of Smart (1972) give a theoretical foundation to Horton's findings. Shreve shows that the most probable network configurations have $R_B$'s in the range encountered in nature, nevertheless to our knowledge no explanation had been offered for the values of $R_A$ displayed by natural basins. We suggest that our analysis explains the value of the ratio $R_B/R_A$ and thus the values of $R_A$.

The above analysis will also suggest that given the small range of variation which is shown by $R_B/R_A$, the controlling parameter in hydrologic similarity is

$$I = \frac{R_L^{0.43}}{L_\Omega} \tag{13}$$

for basins under the same kinematic conditions.

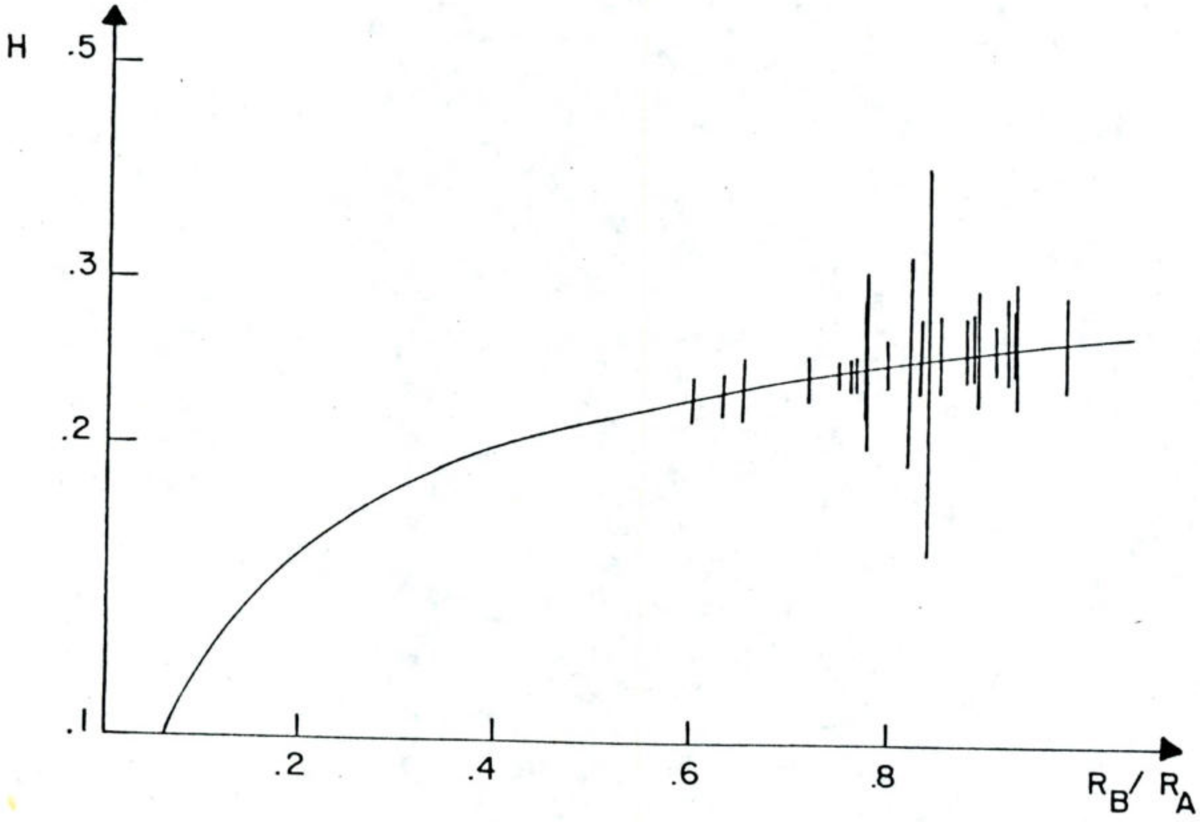


FIGURE 14. Entropy of the IUH as a two states function dependent on $R_B$ / $R_A$.

## FINAL COMMENTS

A link has been established between the geomorphologic structure and the hydrologic response of a basin. This link brings order to the infinite variety of hydrologic responses encountered in nature but obviously this research is just a first step in that direction. Since we are dealing with a feedback system in which hydrology is not only a consequence but also a cause in the geomorphology of a basin, the above link is only the first branch of a loop in the tying together the geomorphologic and hydrologic structure of natural watersheds. The most probable IUH, the one responding to maximum entropy, can be viewed under the constraint of conservation of energy. The IUH itself as a function of velocity can be transformed to a representation of the kinetic energy produced by an unit input of rainfall imposed upon the basin. Since this kinetic energy -now related to geomorphologic parameters- is the result of a potential energy which can also be expressed as function of geomorphology, we want to suggest that the second branch of the loop between hydrology and geomorphology may also be explored under this framework. An example of this mutual influence is the explanation for the values of $R_A$ suggested in this paper by using the geomorphologic IUH. Research in progress at Universidad Simón Bolívar is addressed to the explicit establishment of the second branch of the loop. We wish to close emphazising that the value of the controlled experiments in the analysis of hidrologic similarity is only of a relative character; until the second branch of the loop is established or at least explored in a quantitative manner, the artifitial built-up of basins for purposes of comparing their hydrologic response may conduce to forms and proportions unacceptable to nature and which may still fulfill Horton's laws. This again comes to the point that this research and its companion papers are only a first step in the linking of geomorphologic structures and hydrologic response and that the satisfactory understanding of one branch of the loop is at least partially dependent in the understanding of the other branch.

# REFERENCES

Henderson, F.M., "Some Properties of the Unit Hydrograph", Journal of Geophysical Research, 68(16) pp. 4785-4793, 1963.

Rodríguez-Iturbe, I., J. B. Valdés, G. A. Devoto and Y. Fiallo, "Bases Geomorfológicas para la Hidrología de Cuencas: Informe Final al CONICIT, Graduate Program in Hydrology and Water Resources Publication, Simón Bolívar University, Caracas, Venezuela, September,1978.

Rodríguez-Iturbe, I. and J. B. Valdés, "The Geomorphologic Structure of Hydrologic Response", submitted for publication to Water Resources Research, September 1978.

Schaake, J.C., "Deterministic Urban Runoff Model", in Treatise on Urban Water Systems", edited by M. L. Albertson, L.S. Tucker and D. C. Taylor, Colorado State University, 1971.

Valdés, J.B., Y. Fiallo and I. Rodríguez-Iturbe, "A Rainfall Runnoff Analysis of the Geomorphologic IUH", submitted for publication to Water Resources Research, September 1978.

Yang, C. T., "Potential Energy and Stream Morphology", Water Resources Research, 7(2), pp. 311-322, 1971.

# LIST OF TABLES

# LIST OF FIGURES

| | |
|---|---|
| Figure 10 | IUH's of Basin 1 (------) and 2(———) for different kinematics conditions. |
| Figure 11 | Examples of the IUH's obtained in the second experiment of hydrologic similarity. The theory suggest that the peak and time to peaks of the corresponding pair of hydrographs should be in a 1:2 relationship. |
| Figure 12 | Schematic of Basin 3 used in the third experiment of hydrologic similarity. |
| Figure 13 | IUH's of Basins 1 and 3 which the theory suggest should have the same response function. |
| Figure 14 | Entropy of the IUH as a two states function dependent on $R_B/R_A$. |